

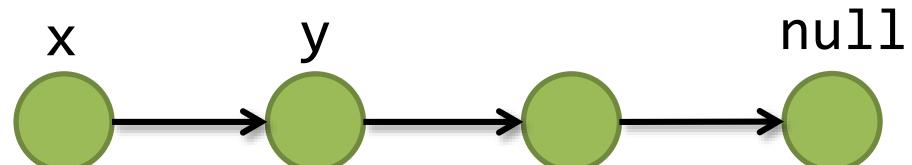
SMT-based Verification of Heap-manipulating Programs

Ruzica Piskac

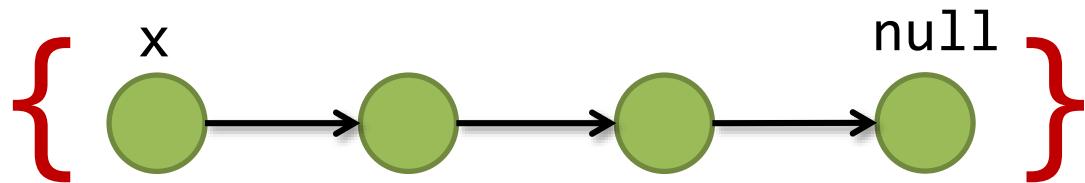
The 11th SRI Summer School on Formal Techniques
May 30 - June 5, 2022

A Motivating Example

```
procedure delete(x: Node)
{
    if (x != null) {
        var y := [x];
        delete(y);
        free(x);
    }
}
```



Proof by Hand-Waving



```
procedure delete(x: Node)
```

```
{
```

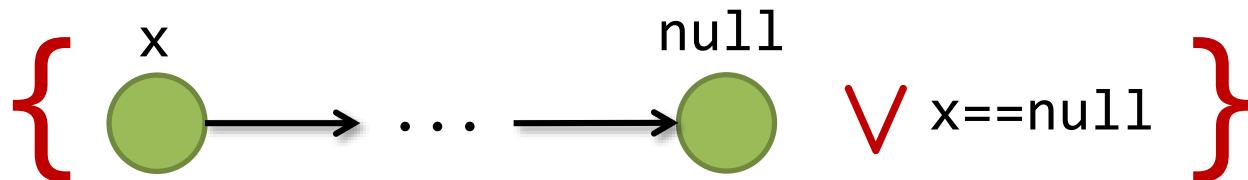
```
    if (x != null) {  
        var y := x.next;  
        delete(y);  
        free(x);
```

```
}
```

```
}
```

```
{ }
```

Proof by Hand-Waving



```
procedure delete(x: Node)
```

```
{
```

```
  if (x != null) {
```

```
    var y := x.next;
```

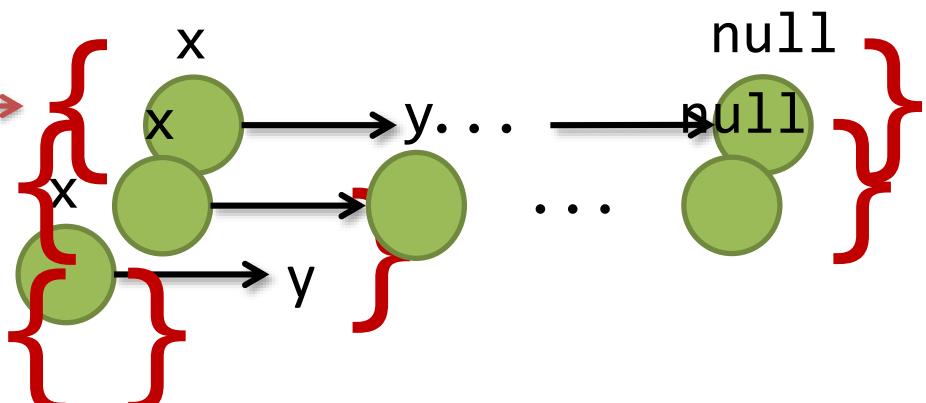
```
    delete(y);
```

```
    free(x);
```

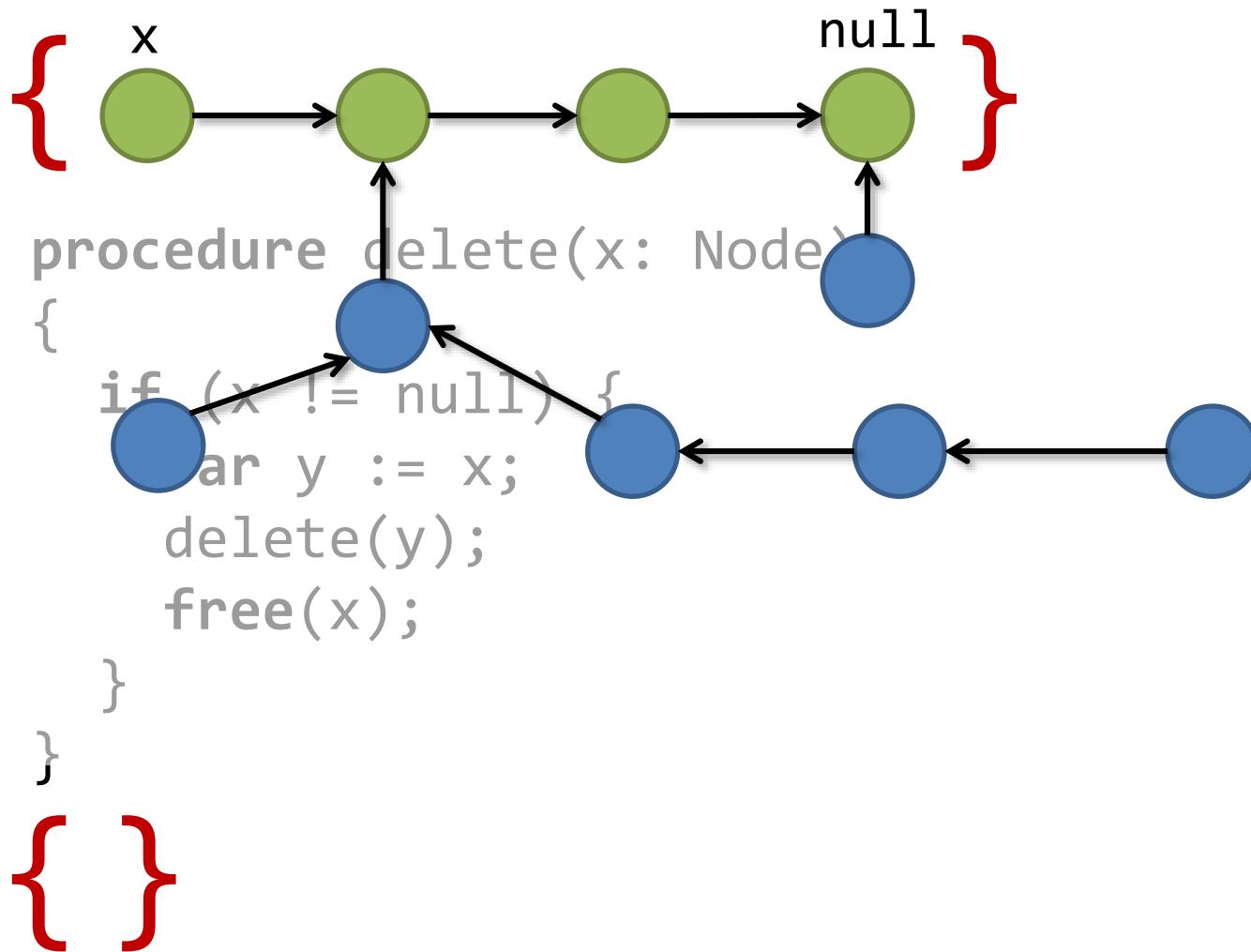
```
}
```

```
}
```

```
{ }
```



Proof by Hand-Waving



Implicit Dynamic Frames

Implicit Dynamic Frames by Example

- Pure assertions

`x.next == y`



Stack

x	10
y	42
...	

Heap

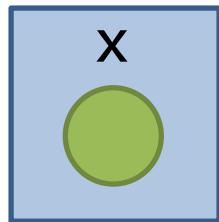
10	42
...	
42	?

```
struct Node {  
    var next: Node;  
}
```

Implicit Dynamic Frames by Example

- Permission predicates

$\text{acc}(x)$



Expresses permission to access (i.e. read/write/deallocate) heap location x.

Assertions describe the program state **and** a set of locations that are allowed to be accessed.

Implicit Dynamic Frames by Example

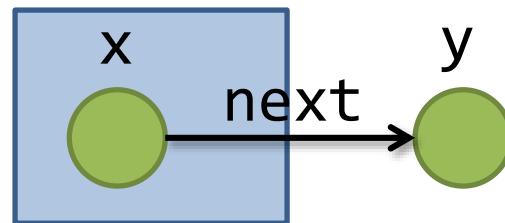
- Separating conjunction

$$\mathbf{acc}(x) * \mathbf{acc}(y)$$


Yields union of permission sets of subformulas.
Permission sets of subformulas must be disjoint.

Implicit Dynamic Frames by Example

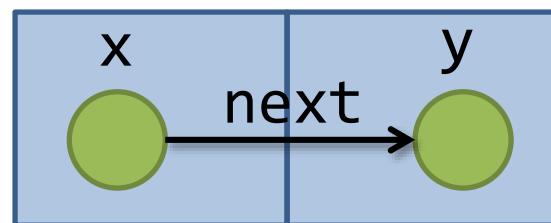
- Separating conjunction

$$\text{acc}(x) * x.\text{next} == y$$


Pure assertions yield no permissions.

Implicit Dynamic Frames by Example

- Separating conjunction

$$\text{acc}(x) * \text{acc}(y) * x.\text{next} == y$$


Implicit Dynamic Frames by Example

- Separating conjunction

$$\text{acc}(x) * \text{acc}(x) * x.\text{next} == y$$

unsatisfiable
?

Implicit Dynamic Frames by Example

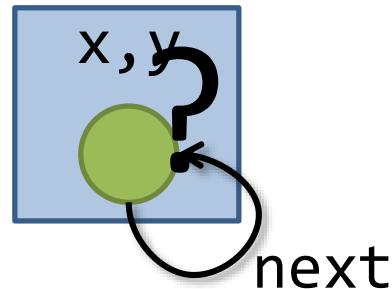
- Classical conjunction

$$\text{acc}(x) \wedge x.\text{next} == y$$

unsatisfiable
?

Implicit Dynamic Frames by Example

- Classical conjunction

$$\text{acc}(x) \wedge \text{acc}(y) * x.\text{next} == y$$


Convention: \wedge has higher precedence than $*$

Syntactic Short-hands

- Empty heap:

$$\text{emp} \equiv (x == x)$$

- Points-to predicates:

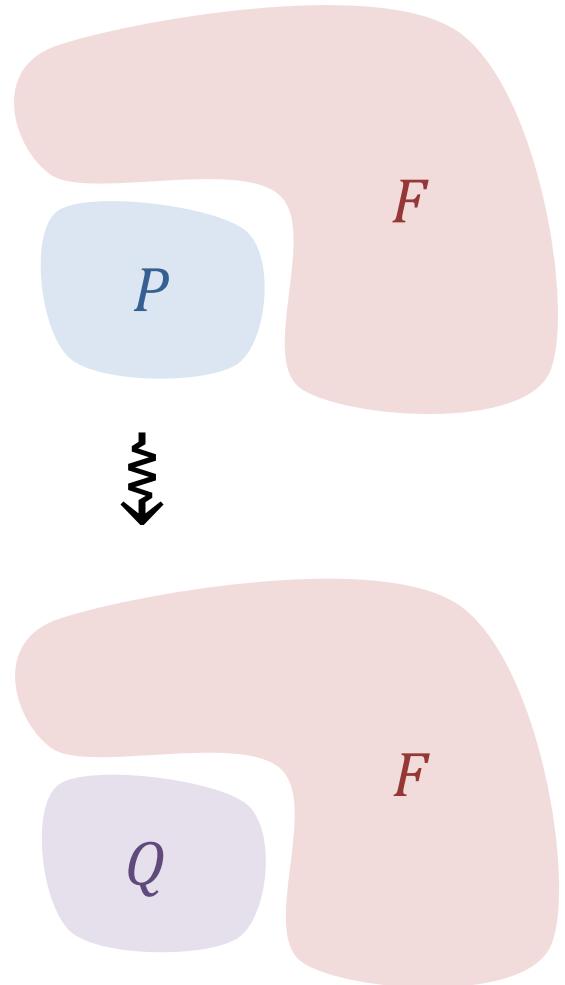
$$x.\text{next} \mapsto y \equiv \text{acc}(x) * x.\text{next} == y$$

Implicit Dynamic Frames: Assertion Semantics

- M : first order structure, D : subset of M 's universe
- $M,D \models P \iff D = \emptyset \text{ and } M \models P \quad \text{if } P \text{ is pure}$
- $M,D \models \mathbf{acc}(t) \iff D = \{M(t)\}$
- $M,D \models P * Q \iff \text{exists } D_1, D_2 \text{ s.t. } D = D_1 \uplus D_2 \text{ and } M,D_1 \models P \text{ and } M,D_2 \models Q$
- $M,D \models P \wedge Q \iff M,D \models P \text{ and } M,D \models Q$
- ... everything else as in classical logic

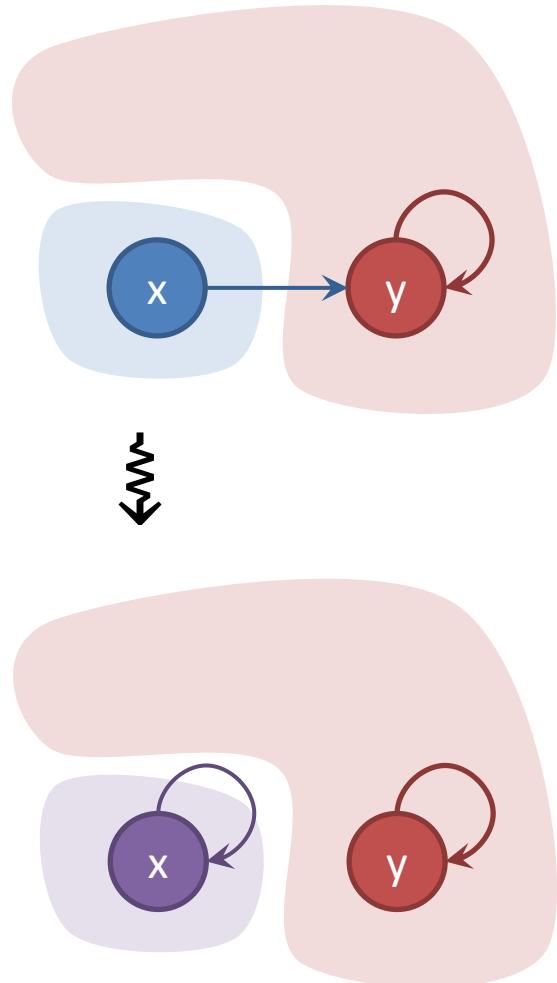
Frame Rule in Separation Logic

$$\frac{\{P\} \text{ C } \{Q\}}{\{P * F\} \text{ C } \{Q * F\}}$$



Frame Rule in Separation Logic

$$\frac{\{x \mapsto y\} \ x.\text{next} := x \ \{x \mapsto x\}}{\{x \mapsto y * y \mapsto y\} \ x.\text{next} := x \ \{x \mapsto x * y \mapsto y\}}$$



Hoare Rules

$\vdash \{ \text{acc}(x) \} x.f := y; \{ x.f \mapsto y \}$

$\vdash \{ x.f \mapsto z \} y := x.f; \{ x.f \mapsto z * y == z \}$

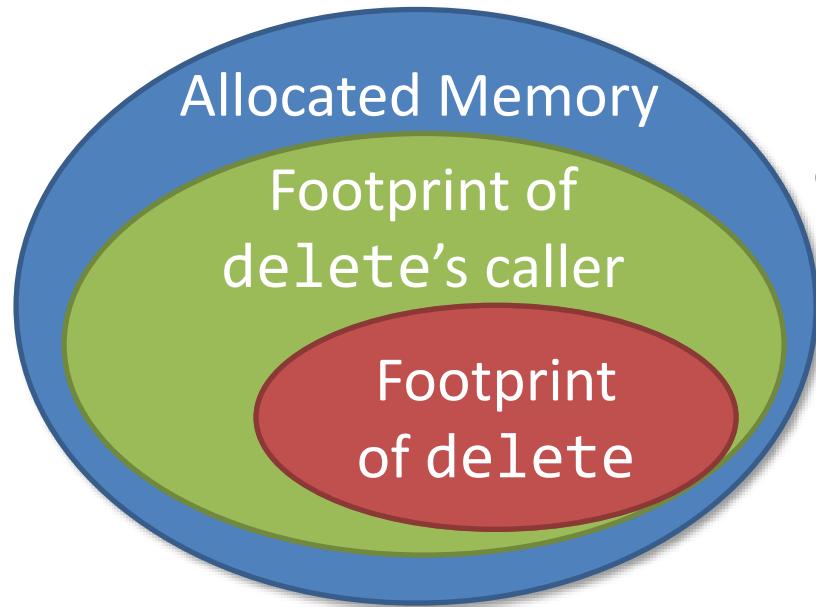
$\vdash \{ \text{emp} \} x := \text{new } T; \{ \text{acc}(x) \}$

$\vdash \{ \text{acc}(x) \} \text{free}(x); \{ \text{emp} \}$

Some Examples of Hoare Triples

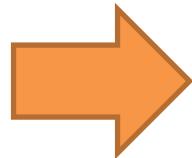
- $\{ \text{acc}(x) \} \quad x.\text{next} := y; \quad \{ \text{acc}(x) * x.\text{next} == y \}$ 
- $\{ \text{acc}(y) \} \quad x.\text{next} := y; \quad \{ \text{acc}(y) * x.\text{next} == y \}$ 
- $\{ \text{emp} \} \quad x := \text{new Node}; \quad \{ \text{acc}(x) \}$ 
- $\{ \text{emp} \} \quad \text{free}(x); \quad \{ \text{emp} \}$ 
- $\{ \text{acc}(x) \} \quad \text{free}(x); \quad \{ \text{emp} \}$ 
- $\{ \text{acc}(x) * \text{acc}(y) * y.\text{next} == z \}$
 $x.\text{next} := y;$
 $\{ \text{acc}(x) * x.\text{next} == y * \text{acc}(y) * y.\text{next} == z \}$ 

Encoding the Frame Rule



before call to `delete`

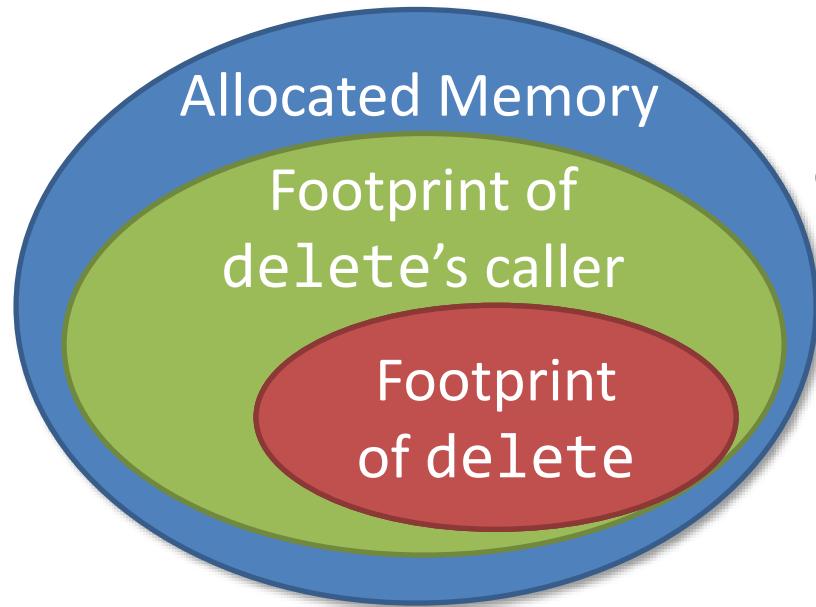
`delete(x);`



?

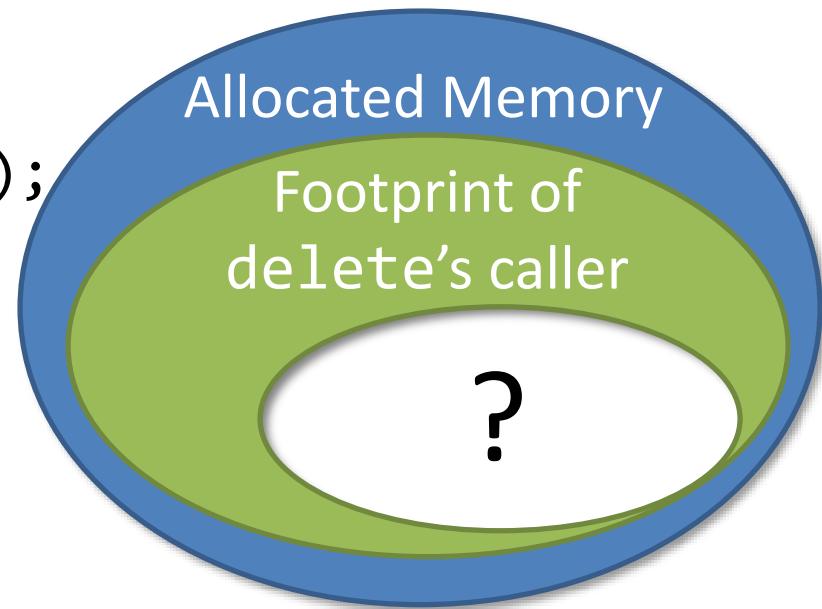
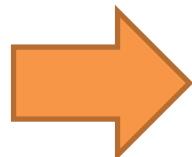
after call to `delete`

Encoding the Frame Rule



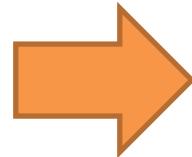
before call to `delete`

`delete(x);`



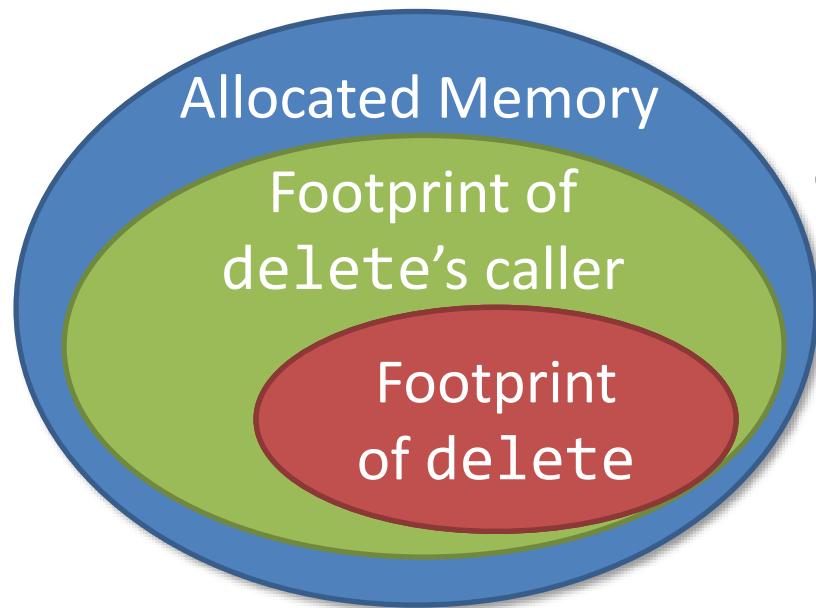
after call to `delete`

`delete(x);`



?

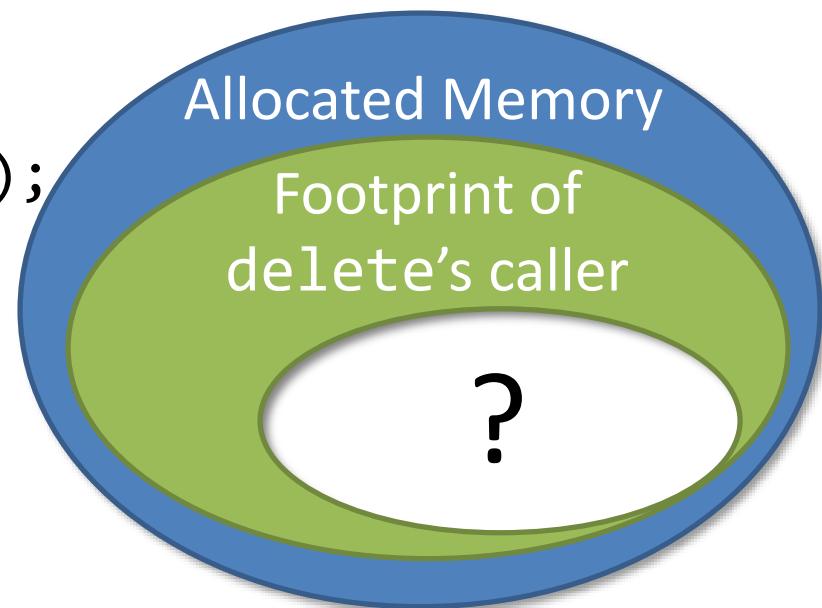
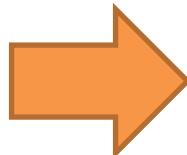
Encoding the Frame Rule



before call to delete

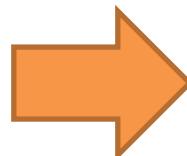


`delete(x);`

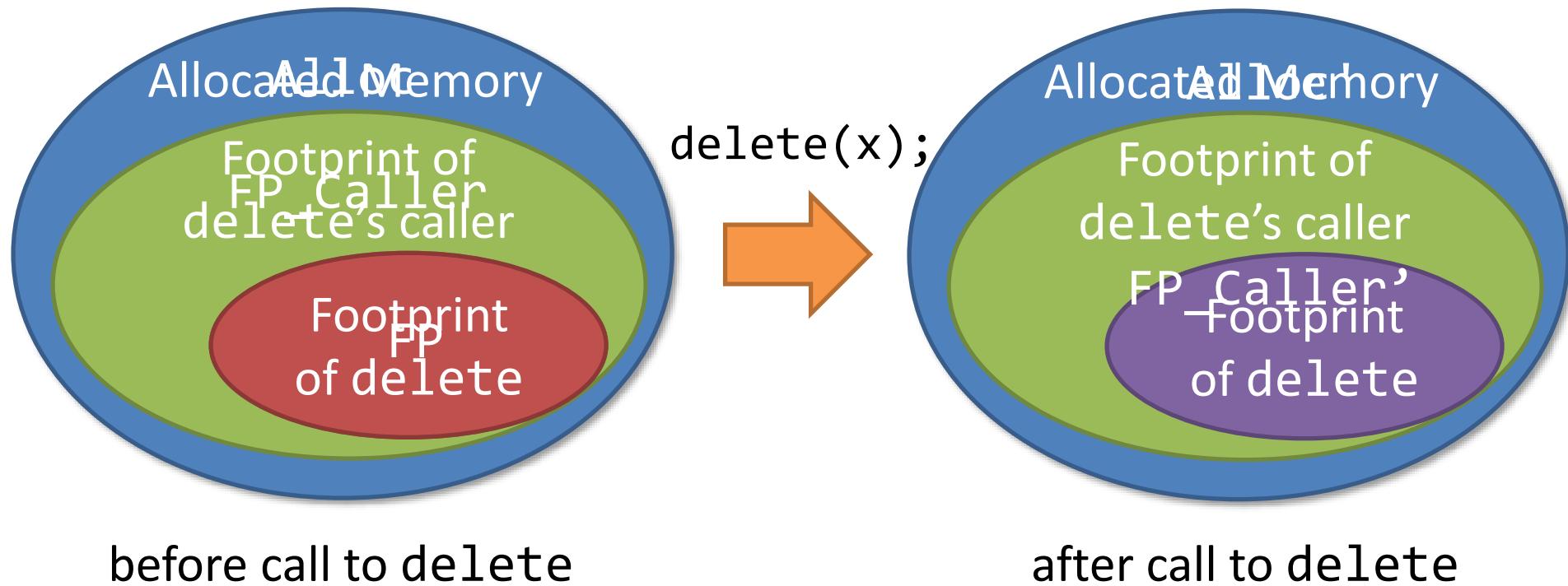


after call to delete

`delete(x);`



Encoding the Frame Rule

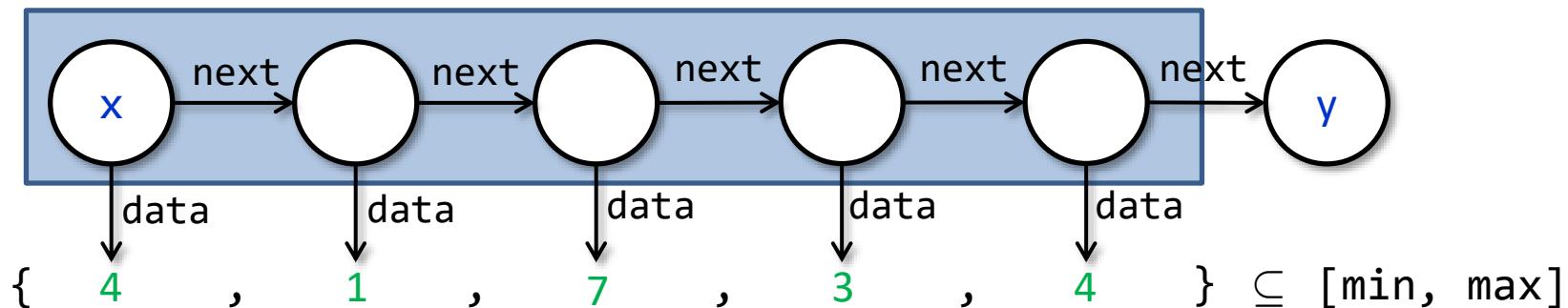


Reasoning about Heap and Data

Inductive Predicates with Data

- bounded list segment

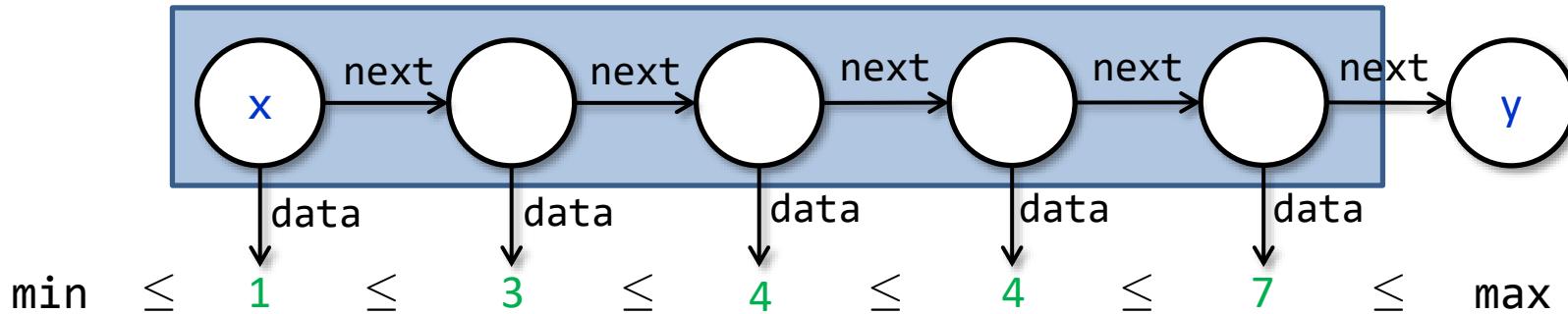
```
bnd_lseg(x, y, min, max) =  
  x = y ∨  
  x ≠ y * acc(x) * min ≤ x.data ≤ max *  
  bnd_lseg(x.next, y, min, max)
```



Inductive Predicates with Data

- sorted list segment

```
srt_lseg(x, y, min, max) =  
  x = y ∨  
  x ≠ y * acc(x) * min ≤ x.data ≤ max *  
  srt_lseg(x.next, y, x.data, max)
```

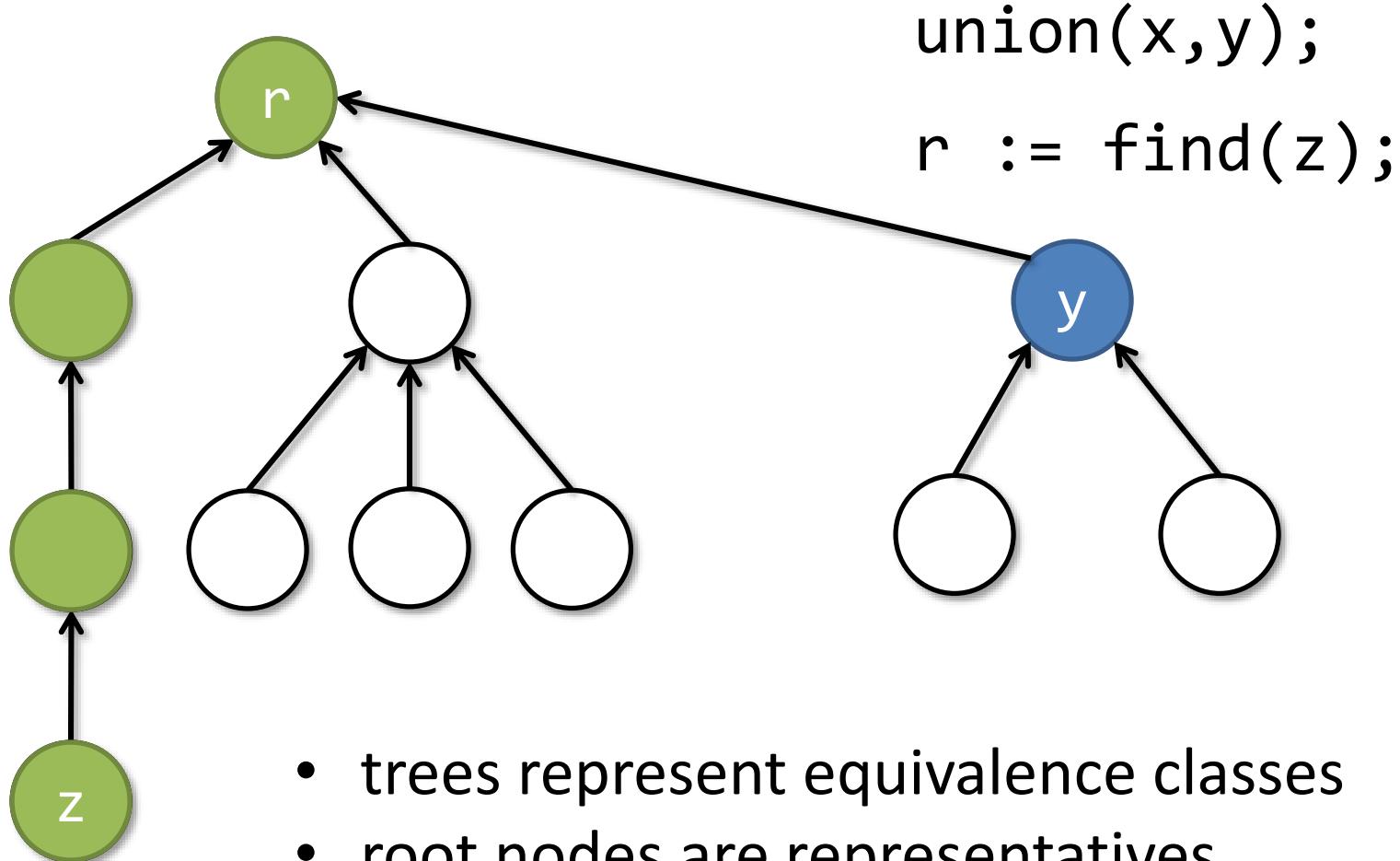


Example: Quicksort

```
procedure quicksort(x: Node, y: Node,
                     ghost min: int, ghost max: int)
  returns (z: Node)
  requires bnd_lseg(x, y, min, max)
  ensures srt_lseg(z, y, min, max)
{
  if (x != y && x.next != y) {
    var p: Node, w: Node;
    z, p := split(x, y, min, max);
    z := quicksort(z, p, min, p.data);
    w := quicksort(p.next, y, p.data, max);
    p.next := w;
  } else z := x;
}
```

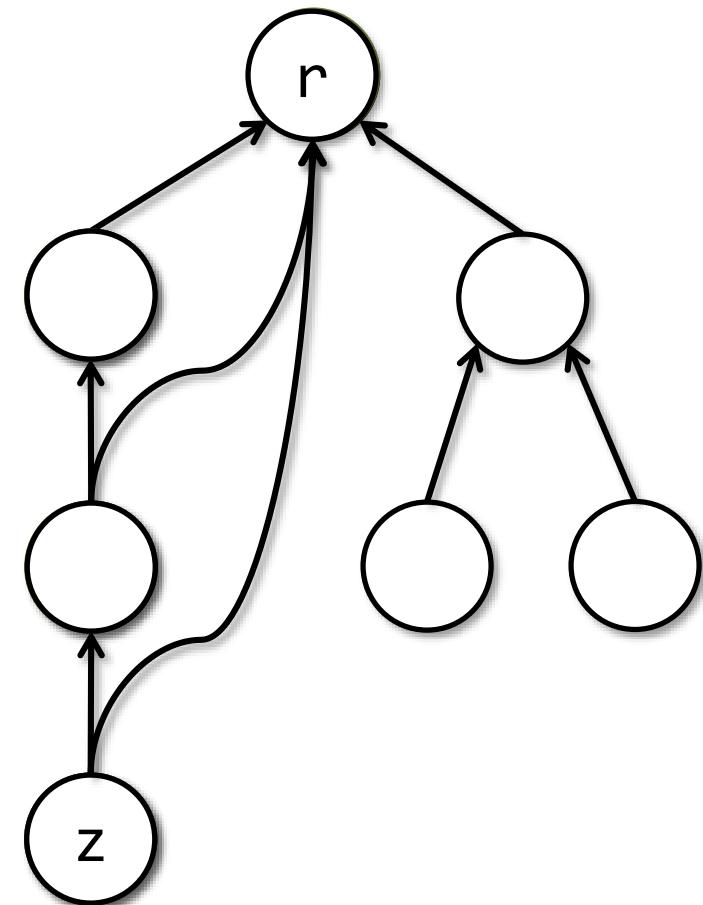
Mixed Specifications

Example: Union/Find Data Structure



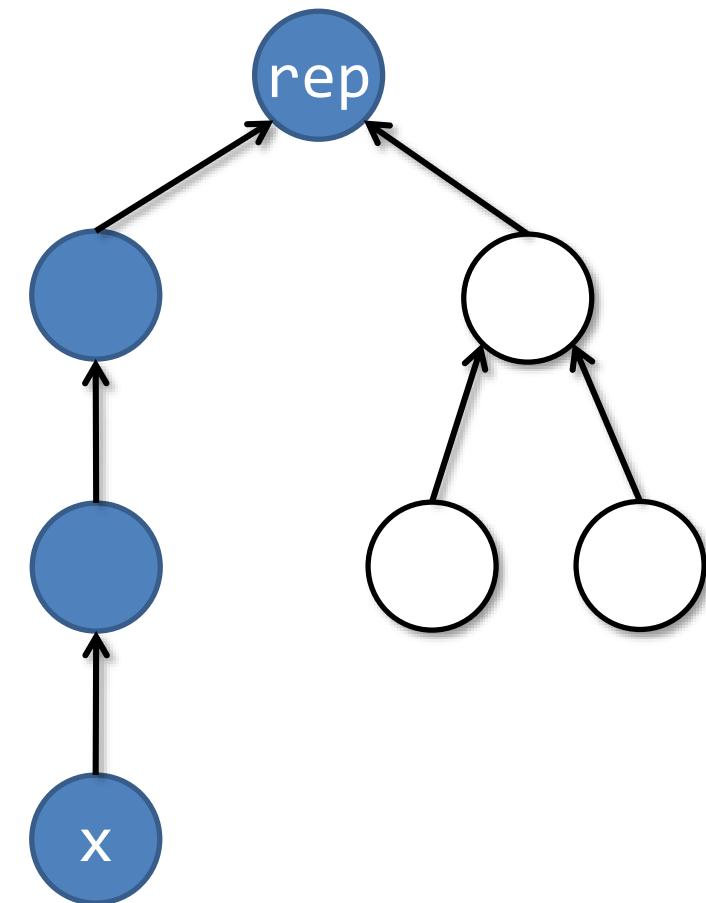
Find with Path Compression

```
procedure find(x: Node)
returns (r: Node)
{
    if (x != null) {
        r := find(x.next);
        x.next := r;
    } else {
        r := x;
    }
}
```



Find with SL Specification

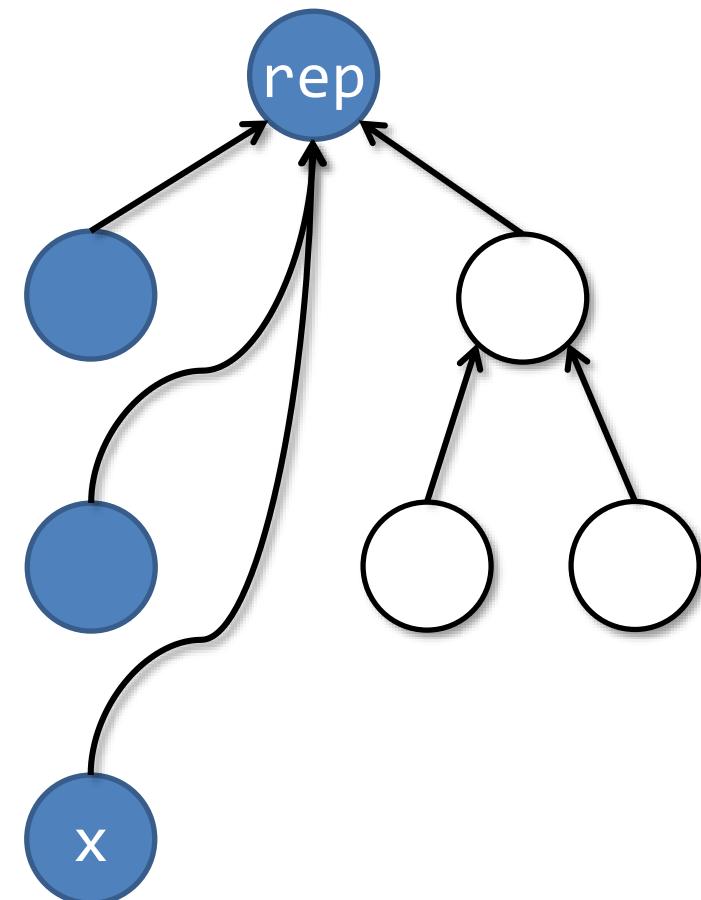
```
procedure find(x: Node, ghost rep: Node)
  returns (r: Node)
  requires lseg(x, rep)
  requires rep.next  $\mapsto$  null
```



Find with SL Specification

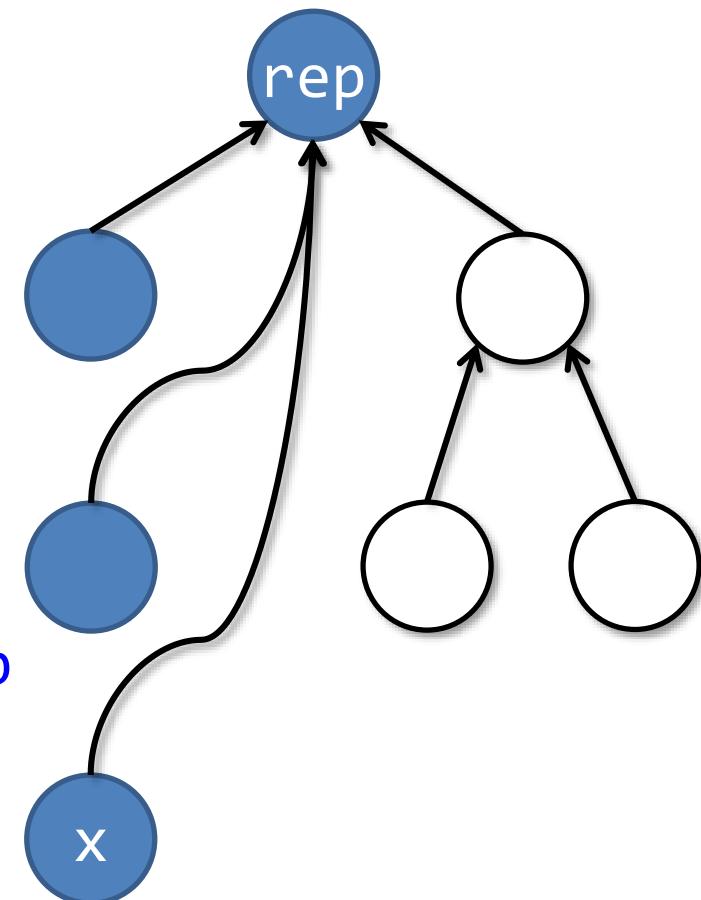
```
procedure find(x: Node, ghost rep: Node)
  returns (r: Node)
  requires rep.next  $\mapsto$  null
  requires lseg(x, rep)
  ensures r == rep
  ensures rep.next  $\mapsto$  null
  ensures ?
```

Postcondition needs to track an unbounded number of list segments.



Find with Mixed Specification

```
procedure find(x: Node, ghost rep: Node,  
implicit ghost X: Set<Node>)  
returns (r: Node)  
requires rep.next  $\mapsto$  null  
requires lseg(x, rep)  $\wedge$  acc(X)  
ensures r == rep  
ensures rep.next  $\mapsto$  null  
ensures acc(X)  
ensures  $\forall z \in X. z.\text{next} == \text{rep}$ 
```



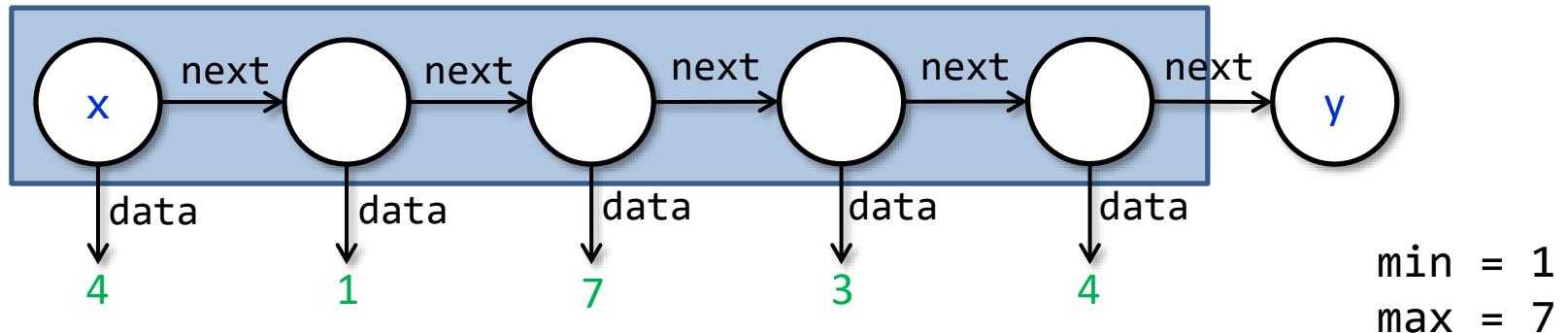
Completeness and Counterexamples

Quicksort Revisited

```
procedure quicksort(x: Node, y: Node,
                     ghost min: int, ghost max: int)
returns (z: Node)
requires bnd_lseg(x, y, min, max)
ensures srt_lseg(z, y, min, max)
{
  if (x != y && x.next != y) {
    var p: Node, w: Node;
    z, p := split(x, y, min, max);
    z := quicksort(z, p, min, p.data);
    w := quicksort(p.next, y, p.data, max);
    p.next := w;
  } else z := x;
}
```

Split with SL Specification

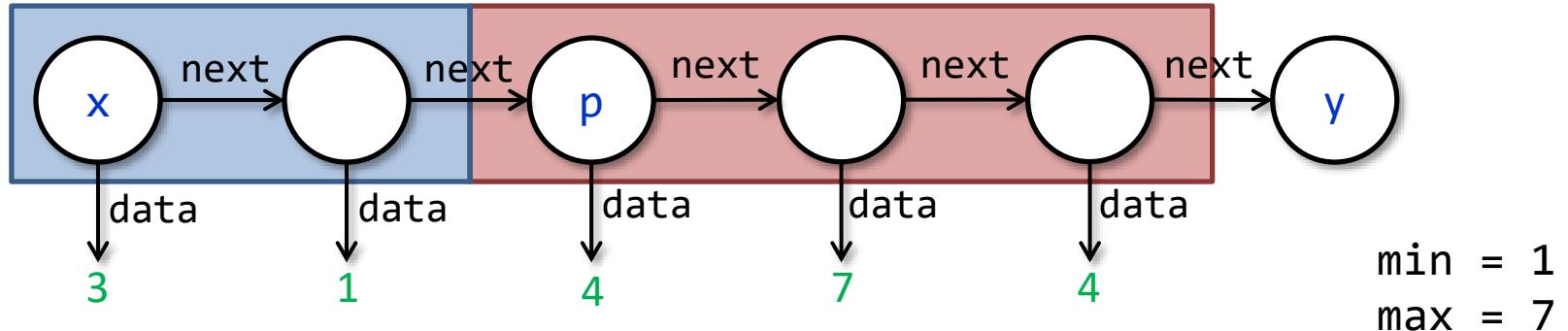
```
procedure split(x: Node, y: Node,
               ghost min: int, ghost max: int)
returns (z: Node, p: Node)
requires bnd_lseg(x, y, min, max) * x ≠ y
ensures bnd_lseg(z, p, min, p.data) *
       bnd_lseg(p, y, p.data, max)
ensures p ≠ y * min ≤ p.data ≤ max
```



Split with SL Specification

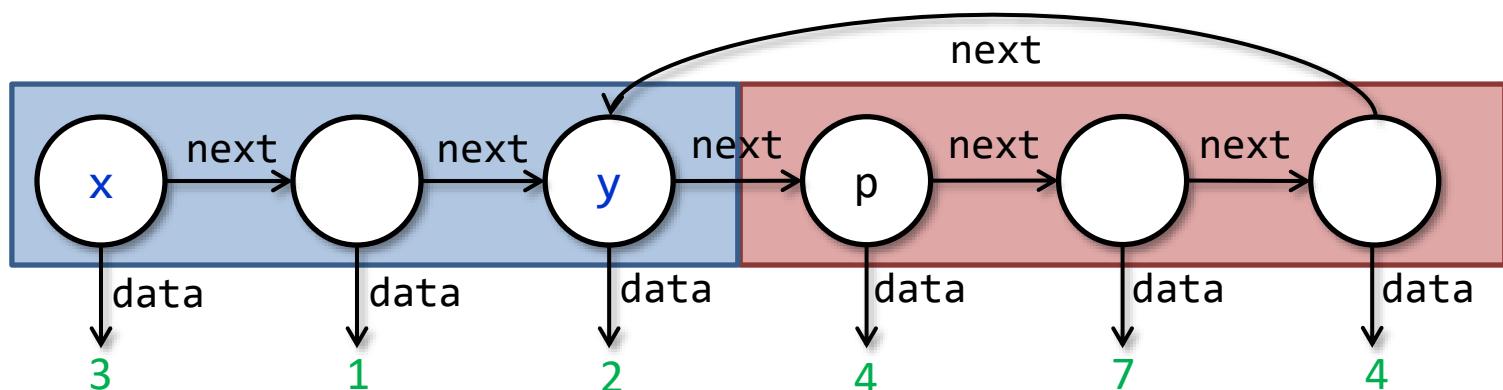
```
procedure split(x: Node, y: Node,  
               ghost min: int, ghost max: int)  
returns (z: Node, p: Node)  
requires bnd_lseg(x, y, min, max) * x ≠ y  
ensures bnd_lseg(z, p, min, p.data) *  
        bnd_lseg(p, y, p.data, max)  
ensures p ≠ y * min ≤ p.data ≤ max
```

free memory



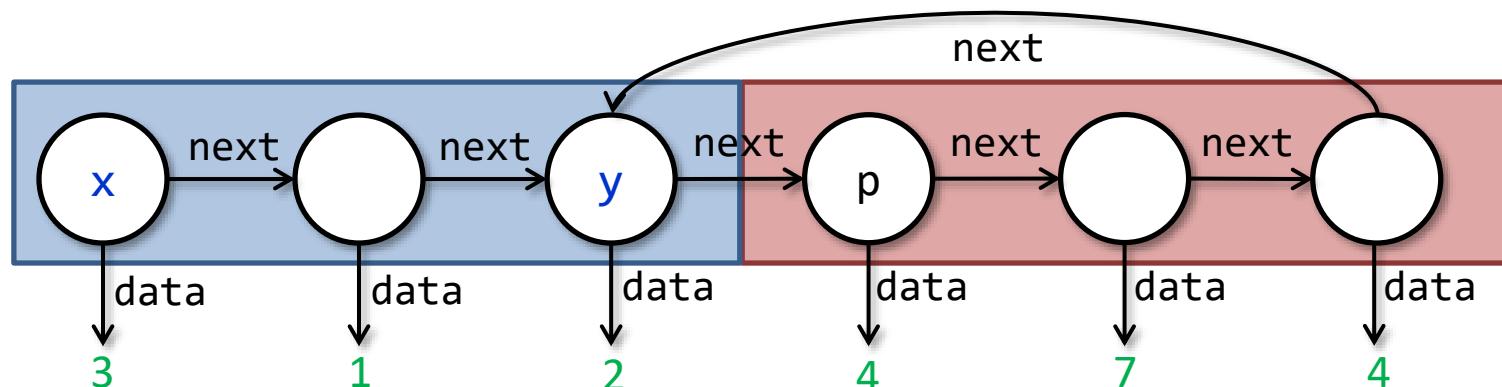
Counterexample for Quicksort Spec.

```
procedure split(x: Node, y: Node,
               ghost min: int, ghost max: int)
returns (z: Node, p: Node)
requires bnd_lseg(x, y, min, max) * x ≠ y
ensures bnd_lseg(z, p, min, p.data) *
       bnd_lseg(p, y, p.data, max)
ensures p ≠ y * min ≤ p.data ≤ max
```



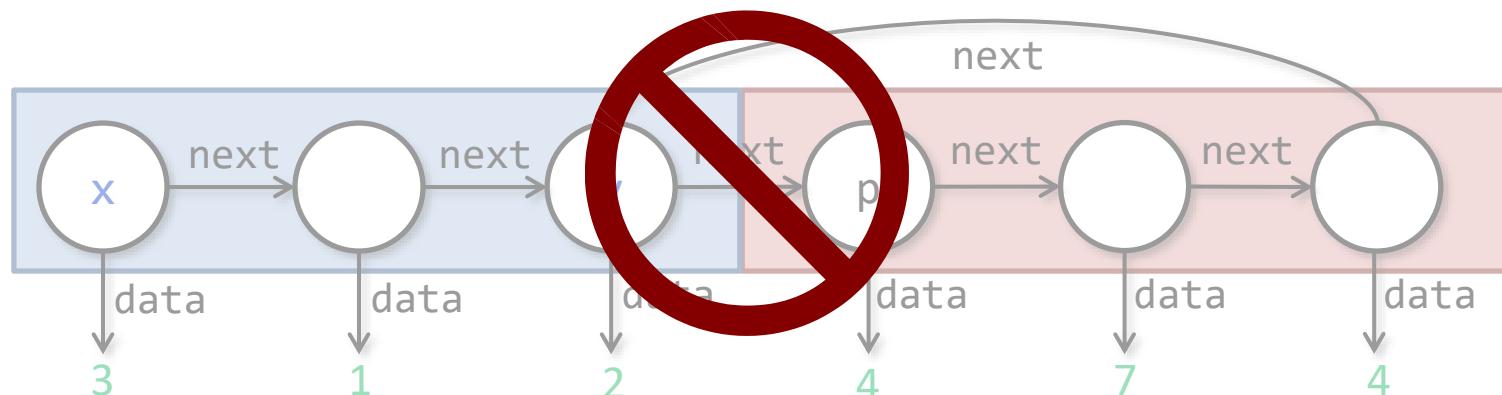
Split with Mixed Specification

```
procedure split(x: Node, y: Node,
               ghost min: int, ghost max: int)
returns (z: Node, p: Node)
requires bnd_lseg(x, y, min, max) * x ≠ y
ensures bnd_lseg(z, p, min, p.data) *
        bnd_lseg(p, y, p.data, max) * Btwn(next,x,p,y)
ensures p ≠ y * min ≤ p.data ≤ max
```



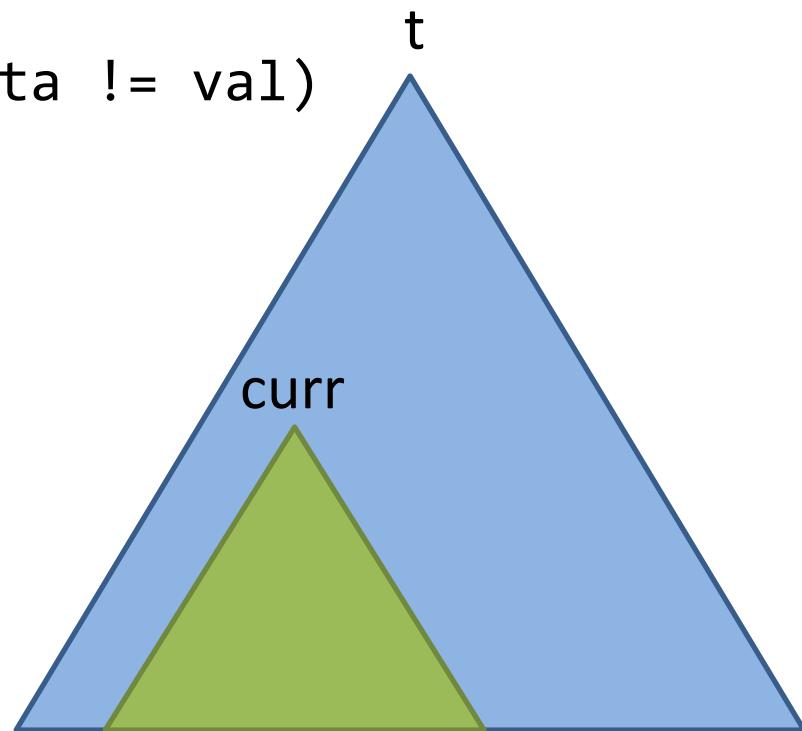
Split with Mixed Specification

```
procedure split(x: Node, y: Node,  
               ghost min: int, ghost max: int)  
returns (z: Node, p: Node)  
requires bnd_lseg(x, y, min, max) * x ≠ y  
ensures bnd_lseg(z, p, min, p.data) *  
        bnd_lseg(p, y, p.data, max) * Btwn(next,x,p,y)  
ensures p ≠ y * min ≤ p.data ≤ max
```



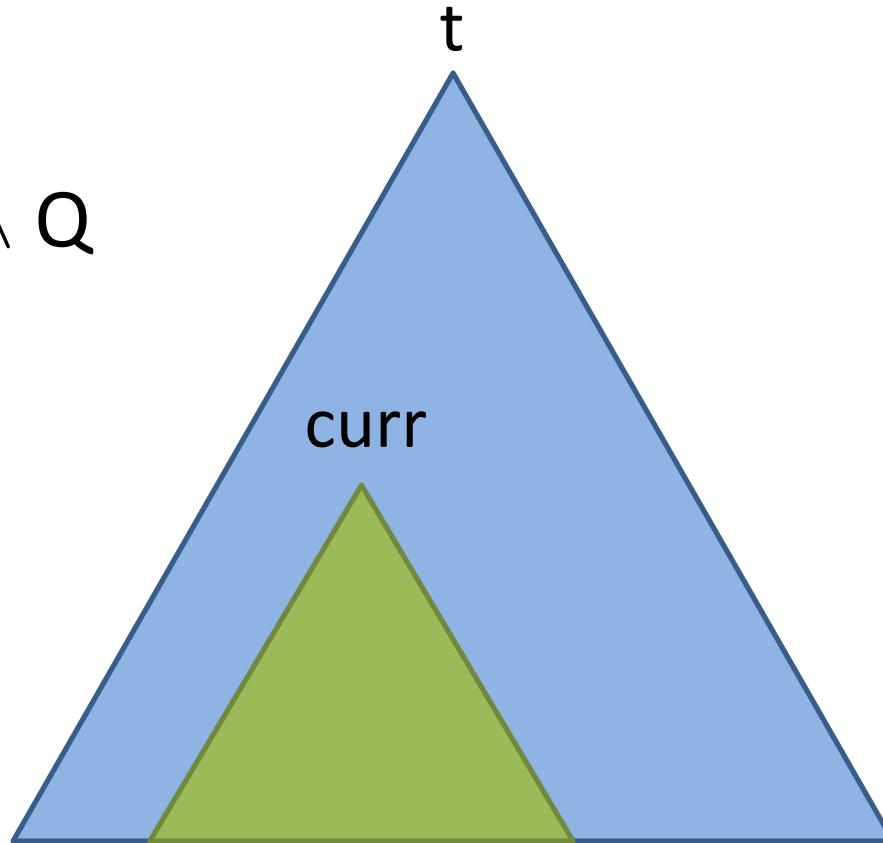
Tail-Recursive Tree Traversal

```
procedure contains(t: Tree, val: Int)
  returns (res: Bool)
  requires tree(t)
  ensures tree(t)
{
  var curr := t;
  while (curr != null && curr.data != val)
    invariant ?
  {
    if (curr.data > val)
      curr := curr.left;
    else if (curr.data < val)
      curr := curr.right;
  }
  return curr != null;
}
```



Poor Man's Magic Wand

$$\begin{aligned} P \text{ --** } Q &\equiv \\ (P * \text{true}) \wedge Q \end{aligned}$$



tree(curr) * (tree(curr) -* tree(t))

Poor Man's Magic Wand

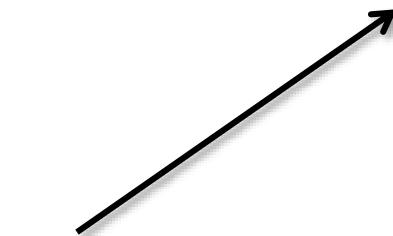
```
procedure contains(t: Tree, val: Int)
  returns (res: Bool)
  requires tree(t)
  ensures tree(t)
{
  var curr := t;
  while (curr != null && curr.data != val)
    invariant tree(curr) -** tree(t)
  {
    if (curr.data > val)
      curr := curr.left;
    else if (curr.data < val)
      curr := curr.right;
  }
  return curr != null;
}
```

Overview of Approach

1. Make frame rule explicit



2. Translate SL assertions to FOL



3. Decide generated VCs



program
+
SL/FOL specs

Program
Transformation

program
+
FOL specs

VC Generator

SMT query

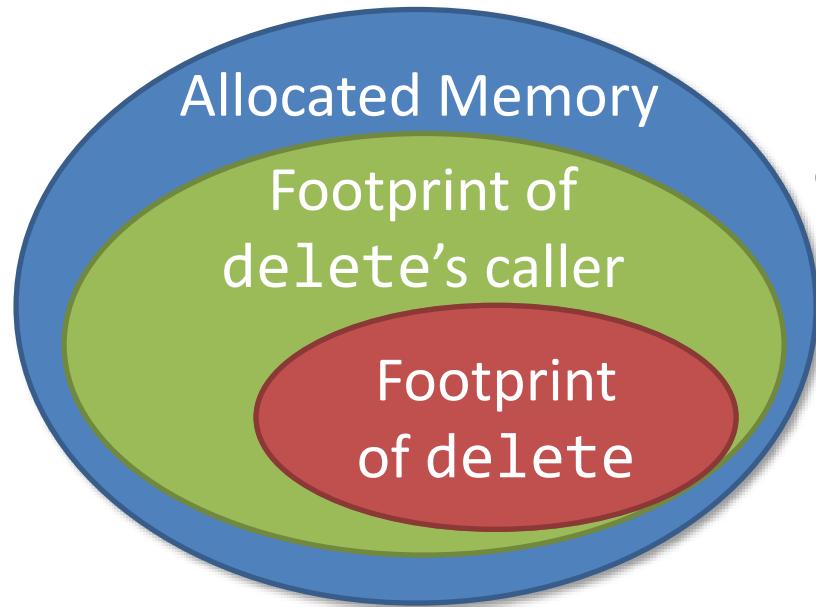
Preprocessing
SMT Solver

Counterexample



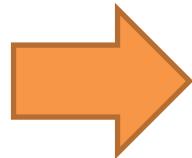
Step 1: Make Frame Rule Explicit

Encoding the Frame Rule



before call to `delete`

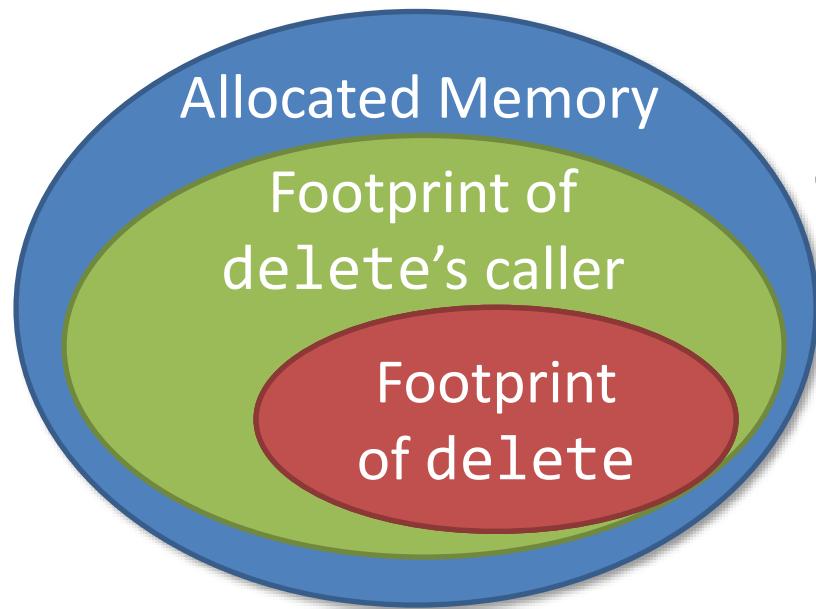
`delete(x);`



?

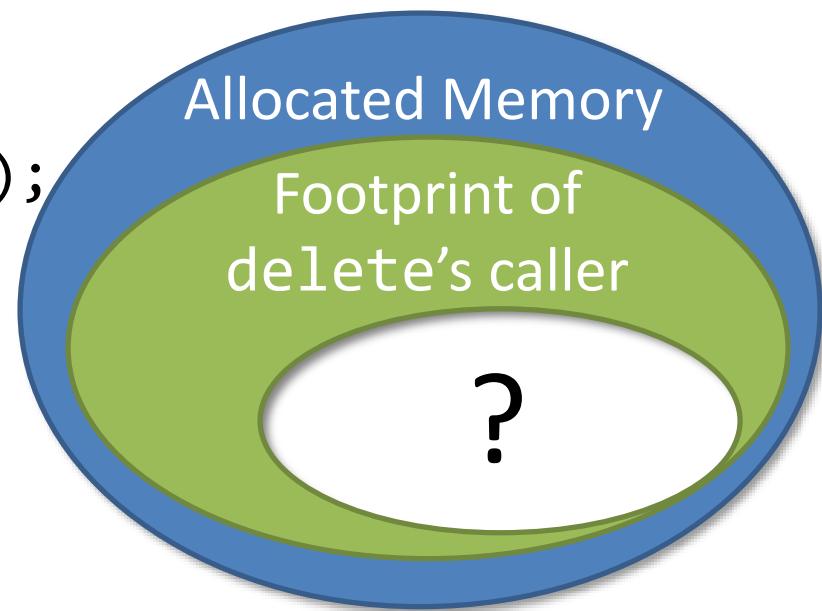
after call to `delete`

Encoding the Frame Rule



before call to `delete`

`delete(x);`

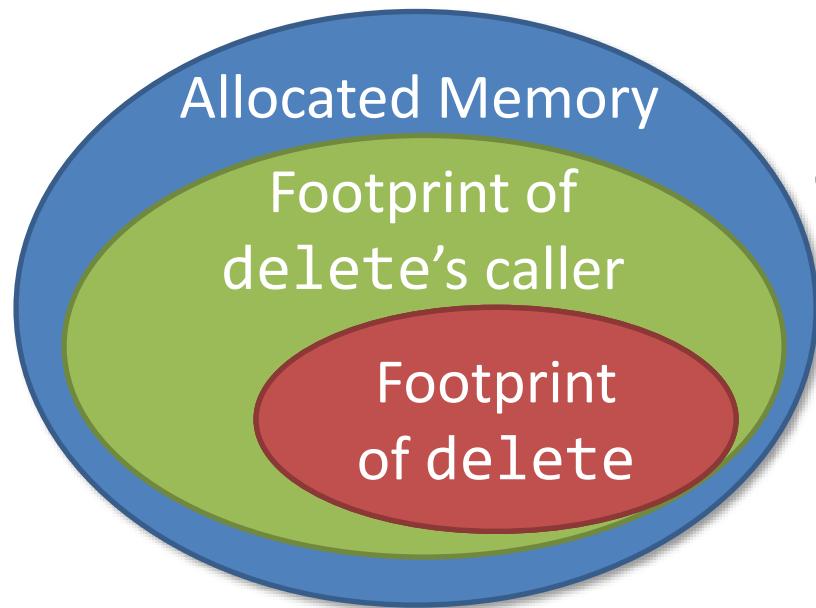


after call to `delete`

`delete(x);`

?

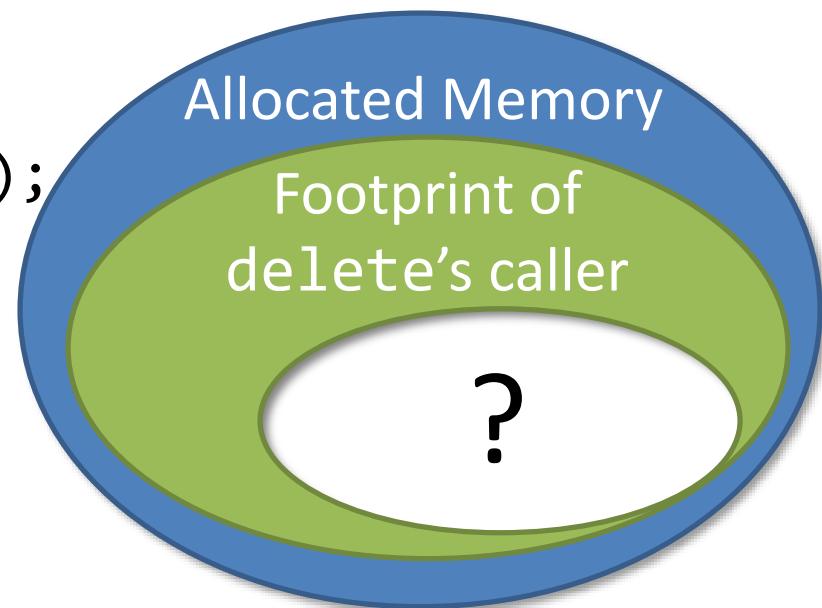
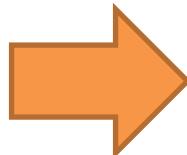
Encoding the Frame Rule



before call to delete

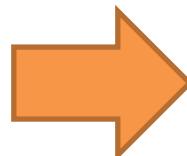


`delete(x);`

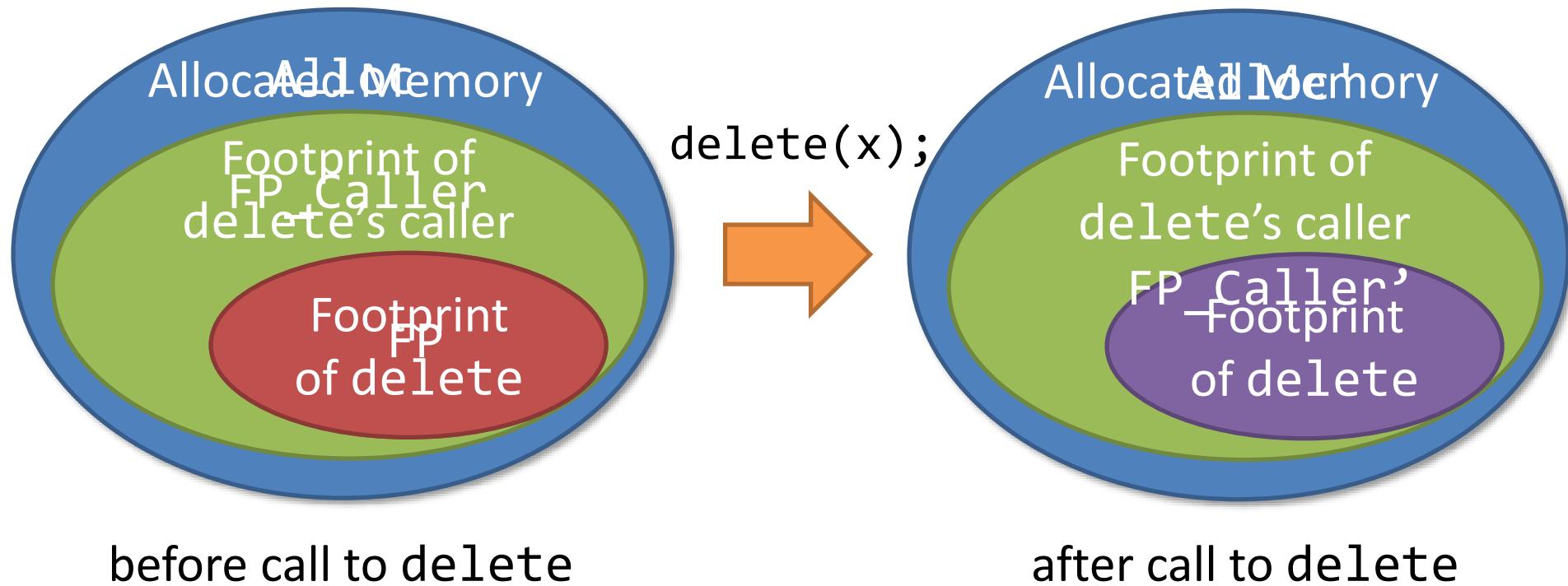


after call to delete

`delete(x);`



Encoding the Frame Rule



Encoding the Frame Rule

```
procedure delete(x: Node
```

```
)
```

```
{
```

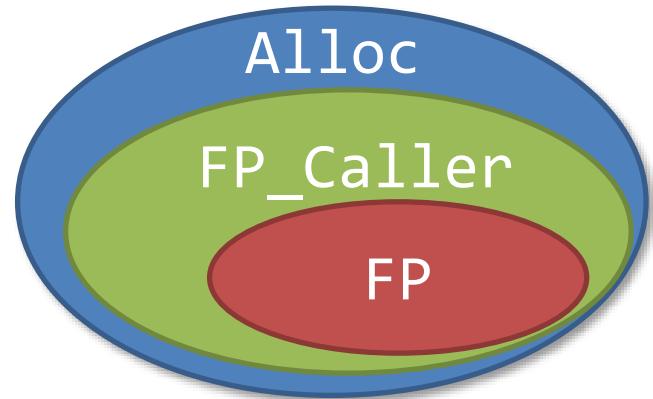
```
    if (x != null) {
```

```
        delete(x.next);
```

```
        free(x);
```

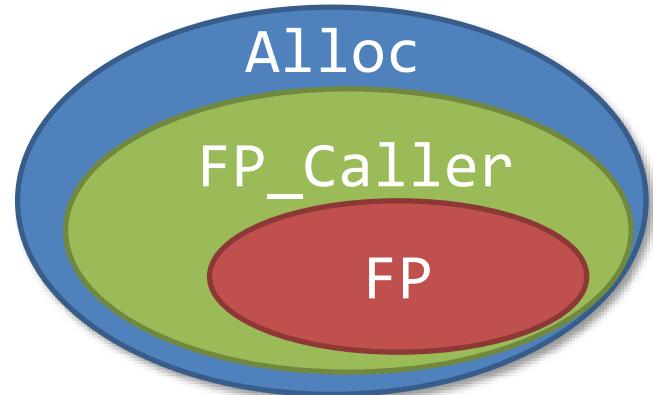
```
}
```

```
}
```



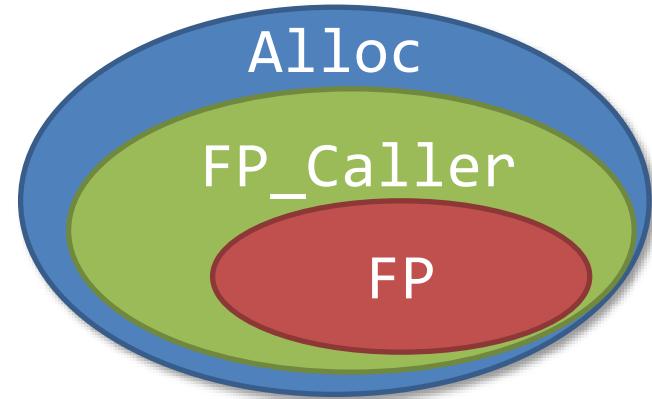
Encoding the Frame Rule

```
ghost var Alloc: Set<Node>;  
  
procedure delete(x: Node,  
                ghost FP_Caller: Set<Node>,  
                implicit ghost FP: Set<Node>)  
returns (ghost FP_Caller': Set<Node>)  
{  
  
    if (x != null) {  
  
        FP := delete(x.next, FP);  
        FP := free(x, FP);  
    }  
  
}
```



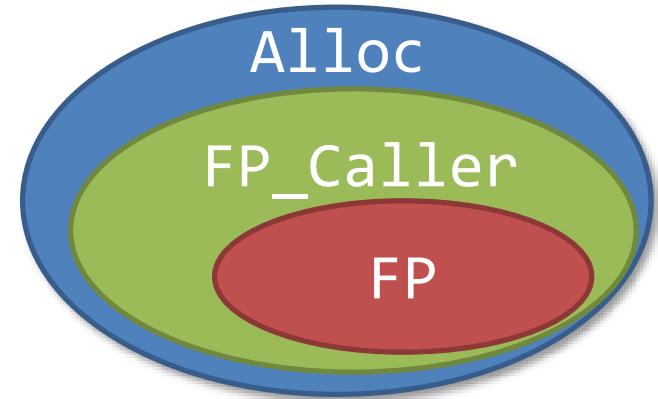
Encoding the Frame Rule

```
ghost var Alloc: Set<Node>;  
  
procedure delete(x: Node)  
    ghost FP_Caller: Set<Node>,  
    implicit ghost FP: Set<Node>  
returns (ghost FP_Caller': Set<Node>)  
{  
    FP_Caller' := FP_Caller \ FP;  
    if (x != null) {  
        FP := delete(x.next, FP);  
        FP := free(x, FP);  
    }  
    FP_Caller' := FP_Caller' ∪ FP;  
}
```



Encoding the Frame Rule

```
ghost var Alloc: Set<Node>;  
  
procedure delete(x: Node)  
    ghost FP_Caller: Set<Node>,  
    implicit ghost FP: Set<Node>  
returns (ghost FP_Caller': Set<Node>)  
{  
    FP_Caller' := FP_Caller \ FP;  
    if (x != null) {  
        pure assert x ∈ FP;  
        FP := delete(x.next, FP);  
        FP := free(x, FP);  
    }  
    FP_Caller' := FP_Caller' ∪ FP;  
}
```



Encoding the Frame Rule

```
procedure delete(x: Node,  
                 ghost FP_Caller: Set<Node>,  
                 implicit ghost FP: Set<Node>)  
returns (ghost FP_Caller': Set<Node>)  
requires lseg(x, null)
```

ensures emp

{ ... }

Encoding the Frame Rule

```
procedure delete(x: Node,  
                 ghost FP_Caller: Set<Node>,  
                 implicit ghost FP: Set<Node>)  
returns (ghost FP_Caller': Set<Node>)  
requires FP ⊆ FP_Caller  
requires Tr(lseg(x,null), FP)  
  
ensures Tr(emp, (Alloc ∩ FP) ∪ (Alloc \ old(Alloc)))
```

{ ... }

Encoding the Frame Rule

```
procedure delete(x: Node,  
                 ghost FP_Caller: Set<Node>,  
                 implicit ghost FP: Set<Node>)  
returns (ghost FP_Caller': Set<Node>)
```

```
requires FP ⊆ FP_Caller
```

```
requires FP ⊆ FP_Caller'
```

```
free ensures FP_Caller' ⊆ Alloc
```

```
free ensures FP ⊆ Alloc
```

```
ensures FP_Caller' ⊆ Alloc
```

```
free ensures FP ⊆ Alloc
```

```
free ensures FP_Caller' ⊆ Alloc
```

```
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```

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```

```
ensures FP_Caller' ⊆ Alloc
```

```
free ensures FP ⊆ Alloc
```

```
ensures FP_Caller' ⊆ Alloc
```

```
{ ... }
```

Encoding is inspired by **implicit dynamic frames**

[Smans, Jacobs, Piessens, 2008]

Used, e.g., in the **VeriCool** and **Chalice** tools

$$(Alloc \cap FP) \cup (\text{Alloc} \setminus \text{old}(Alloc))$$

Encoding the Frame Rule

```
procedure delete(x: Node,  
                ghost FP_Caller: Set<Node>,  
                implicit ghost FP: Set<Node>)  
returns (ghost FP_Caller': Set<Node>)  
requires FP ⊆ FP_Caller  
requires Tr(lseg(x,null), FP) ← The secret sauce  
free requires FP_Caller ⊆ Alloc  
free requires null ∉ Alloc  
ensures Tr(emp, (Alloc ∩ FP) ∪ (Alloc \ old(Alloc)))  
free ensures Frame(old(Alloc), FP, old(next), next)  
free ensures FP_Caller' = (FP_Caller \ FP) ∪  
                           (Alloc ∩ FP) ∪ (Alloc \ old(Alloc))  
free ensures FP_Caller' ⊆ Alloc  
free ensures null ∉ Alloc  
{ ... }
```

Step 2: Translating SL Assertions

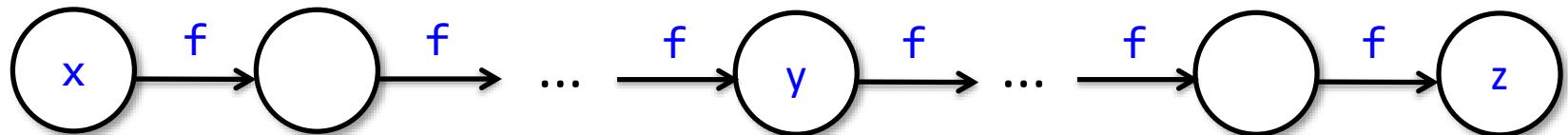
Target of Translation: GRASS (Graph Reachability and Stratified Sets)

- Theory of Reachability in Mutable Graphs
 - encodes structure of the heap
(inductive predicates)
- Theory of Stratified Sets
 - encodes frame rule / separating conjunction

Reachability in Mutable Function Graphs

(Extension of [Nelson POPL'83], [Lahiri, Qadeer POPL'08])

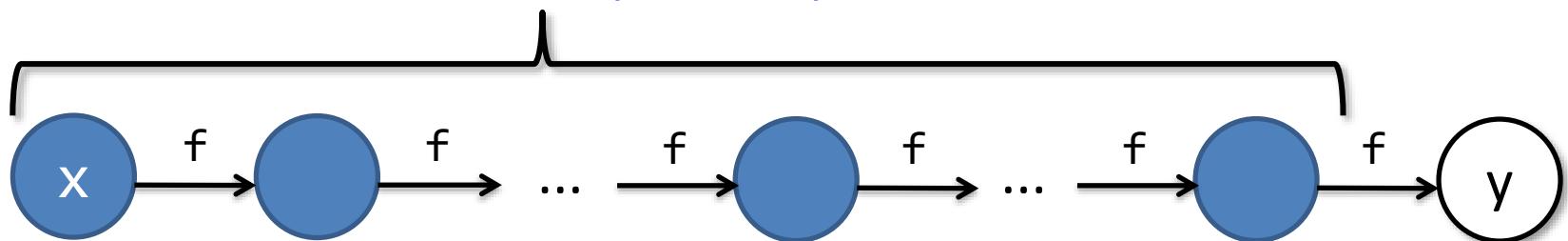
- $\text{sel}(f, x)$ field access $x.f$
- $\text{upd}(f, x, y)$ field update $f[x := y]$
- $\text{Btwn}(f, x, y, z)$ reachability $x \xrightarrow{f} y \xrightarrow{f} z$



$\text{Btwn}(f, x, y, z)$ means z is reachable from x via f and y is on the shortest path between x and z

Stratified Sets

- operations: $X \cup Y, X \cap Y, X \setminus Y, \dots$
- predicates: $x \in X, X \subseteq Y, X = Y$
- literals: $\{ x :: P(x) \}$
 - Examples:
 - $\{ z :: z = x \}$
 - $\{ z :: \text{Btwn}(f, x, z, y) \wedge z \neq y \}$



Translating SL Assertions to GRASS

- $\text{Tr}(\text{emp}, X) \equiv X = \emptyset$
- $\text{Tr}(\text{acc}(t), X) \equiv X = t$
- $\text{Tr}(F, X) \equiv F \wedge X = \emptyset \quad \text{if } F \text{ is pure}$
- $\text{Tr}(\text{Iseg}(x,y), X) \equiv \text{Btwn}(\text{next}, x, y, y) \wedge X = \{z :: \text{Btwn}(\text{next}, x, z, y) \wedge z \neq y\}$
- $\text{Tr}(F * G, X) \equiv \exists Y, Z :: \text{Tr}(F, Y) \wedge \text{Tr}(G, Z) \wedge X = Y \uplus Z$
- $\text{Tr}(F -** G, X) \equiv \exists Y :: \text{Tr}(F, Y) \wedge \text{Tr}(G, X) \wedge Y \subseteq X$
- $\text{Tr}(F \wedge G, X) \equiv \text{Tr}(F, X) \wedge \text{Tr}(G, X)$
- $\text{Tr}(\neg F, X) \equiv \neg \text{Tr}(F, X)$

Example: Delete

```
procedure delete(x: Node,
                 ghost FP_Caller: Set<Node>,
                 implicit ghost FP: Set<Node>)
returns (ghost FP_Caller': Set<Node>)
requires FP ⊆ FP_Caller
requires Tr(lseg(x,null), FP)
free requires FP_Caller ⊆ Alloc
free requires null ∉ Alloc
ensures Tr(emp, (Alloc ∩ FP) ∪ (Alloc \ old(Alloc)))
free ensures Frame(old(Alloc), FP, old(next), next)
free ensures FP_Caller' = (FP_Caller \ FP) ∪
                           (Alloc ∩ FP) ∪ (Alloc \ old(Alloc))
free ensures FP_Caller' ⊆ Alloc
free ensures null ∉ Alloc
{ ... }
```

Example: Delete

```
procedure delete(x: Node,
                ghost FP_Caller: Set<Node>,
                implicit ghost FP: Set<Node>)
returns (ghost FP_Caller': Set<Node>)
requires FP ⊆ FP_Caller
requires Btwn(next,x,y,y) ∧ FP = {z. Btwn(next,x,z,y) ∧ z ≠ y}
free requires FP_Caller ⊆ Alloc
free requires null ∈ Alloc
ensures (Alloc ∩ FP) ∪ (Alloc \ old(Alloc)) = ∅
free ensures Frame(old(Alloc), FP, old(next), next)
free ensures FP_Caller' = (FP_Caller \ FP) ∪
                           (Alloc ∩ FP) ∪ (Alloc \ old(Alloc))
free ensures FP_Caller' ⊆ Alloc
free ensures null ∈ Alloc
{ ... }
```

Step 3: Deciding GRASS

Dealing with Second-Order Quantifiers

- $\text{Tr}(\text{emp}, X) \equiv X = \emptyset$
 - $\text{Tr}(\text{acc}(t), X) \equiv X = t$
 - $\text{Tr}(F, X) \equiv F \wedge X = \emptyset$ if F is pure
 - $\text{Tr}(\text{Iseg}(x,y), X) \equiv \text{Btwn}(\text{next}, x, y, y) \wedge X = \{z :: \text{Btwn}(\text{next}, x, z, y) \wedge z \neq y\}$
 - $\text{Tr}(F * G, X) \equiv \exists Y, Z :: \text{Tr}(F, Y) \wedge \text{Tr}(G, Z) \wedge X = Y \uplus Z$
 - $\text{Tr}(F -** G, X) \equiv \exists Y :: \text{Tr}(F, Y) \wedge \text{Tr}(G, X) \wedge Y \subseteq X$
 - $\text{Tr}(F \wedge G, X) \equiv \text{Tr}(F, X) \wedge \text{Tr}(G, X)$
 - $\text{Tr}(\neg F, X) \equiv \neg \text{Tr}(F, X)$
- Permission sets are uniquely determined by formula structure
- Quantifiers can be eliminated

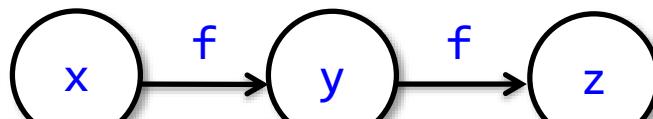
First-Order Axioms for Btwn

- $\forall f x. \text{Btwn}(f, x, x, x)$
- $\forall f x. \text{Btwn}(f, x, x.f, x.f)$
- $\forall f x y. \text{Btwn}(f, x, y, y) \Rightarrow x = y \vee \text{Btwn}(f, x, x.f, y)$
- $\forall f x y. x.f = x \wedge \text{Btwn}(f, x, y, y) \Rightarrow x = y$
- $\forall f x y. \text{Btwn}(f, x, y, x) \Rightarrow x = y$
- $\forall f x y z. \text{Btwn}(f, x, y, y) \wedge \text{Btwn}(f, y, z, z) \Rightarrow \text{Btwn}(f, x, y, z) \vee \text{Btwn}(f, x, z, y)$
- $\forall f x y z. \text{Btwn}(f, x, y, z) \Rightarrow \text{Btwn}(f, x, y, y) \wedge \text{Btwn}(f, y, z, z)$
- $\forall f x y z. \text{Btwn}(f, x, y, y) \wedge \text{Btwn}(f, y, z, z) \Rightarrow \text{Btwn}(f, x, z, z)$
- $\forall f x y z u. \text{Btwn}(f, x, y, z) \wedge \text{Btwn}(f, y, u, z) \Rightarrow \text{Btwn}(f, x, u, z) \wedge \text{Btwn}(f, x, y, u)$
- $\forall f x y z u. \text{Btwn}(f, x, y, z) \wedge \text{Btwn}(f, x, u, y) \Rightarrow \text{Btwn}(f, x, u, z) \wedge \text{Btwn}(f, y, u, z)$

But I thought transitive closure was not first-order definable!?

Completeness of Axioms for Btwn

- A model of $\text{Btwn}(f,x,y,z)$

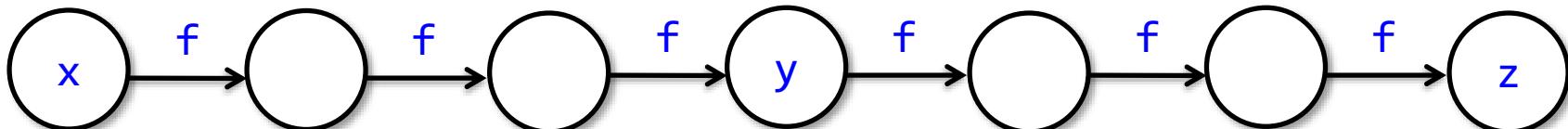


There are arbitrarily large finite models

+

Compactness Theorem \Rightarrow there must also be infinite models

- and another

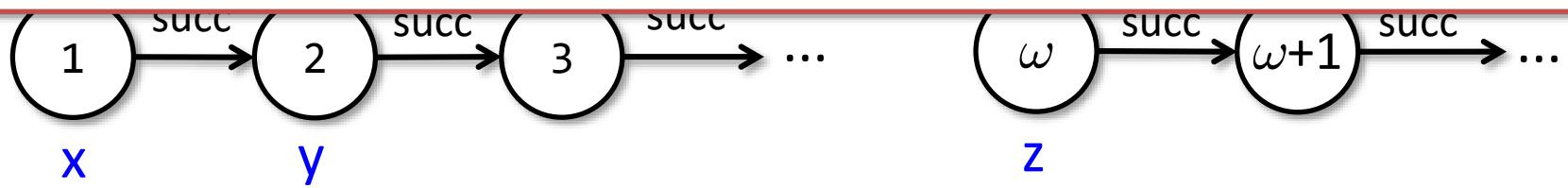


A Degenerated Infinite Model M of $\text{Btwn}(f,x,y,z)$

- $M = \text{ordinal numbers}$
- $M(f) = \text{succ}$

Completeness of first-order axioms for Btwn :

- Only infinite models can be degenerated
- If there is a model, then there is also a finite one



\leq is not succ^*

First-Order Axioms for Btwn

- $\forall f x. \text{Btwn}(f, x, x, x)$
- $\forall f x. \text{Btwn}(f, x, \text{sel}(f, x), \text{sel}(f, x))$
- $\forall f x y. \text{Btwn}(f, x, y, y) \Rightarrow x = y \vee \text{Btwn}(f, x, \text{sel}(f, x), y)$
- $\forall f x y. \text{sel}(f, x) = x \wedge \text{Btwn}(f, x, y, y) \Rightarrow x = y$
- $\forall f x y. \text{Btwn}(f, x, y, x) \Rightarrow x = y$
- $\forall f x y z. \text{Btwn}(f, x, y, y) \wedge \text{Btwn}(f, y, z, z) \Rightarrow \text{Btwn}(f, x, y, z) \vee \text{Btwn}(f, x, z, y)$
- $\forall f x y z. \text{Btwn}(f, x, y, z) \Rightarrow \text{Btwn}(f, x, y, y) \wedge \text{Btwn}(f, y, z, z)$
- $\forall f x y z. \text{Btwn}(f, x, y, y) \wedge \text{Btwn}(f, y, z, z) \Rightarrow \text{Btwn}(f, x, z, z)$
- $\forall f x y z u. \text{Btwn}(f, x, y, z) \wedge \text{Btwn}(f, y, u, z) \Rightarrow \text{Btwn}(f, x, u, z) \wedge \text{Btwn}(f, x, y, u)$
- $\forall f x y z u. \text{Btwn}(f, x, y, z) \wedge \text{Btwn}(f, x, u, z) \Rightarrow \text{Btwn}(f, x, z) \wedge \text{Btwn}(f, y, u, z)$

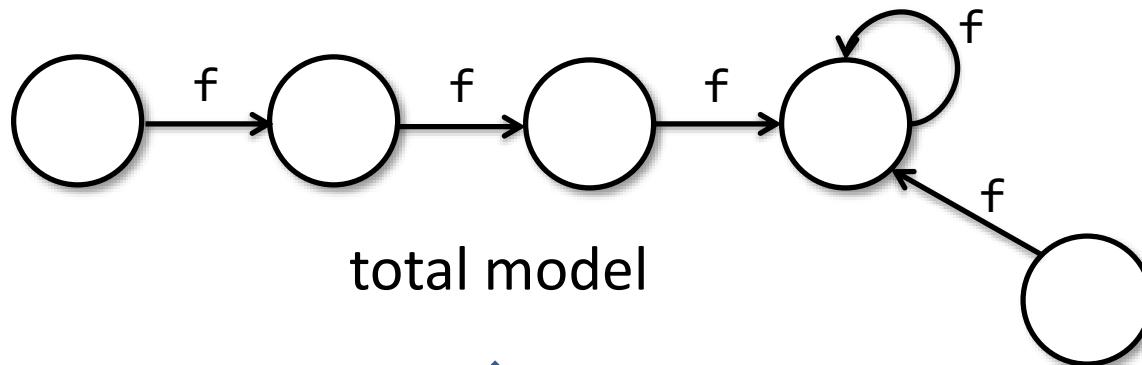
Almost in EPR!

Need to consider more general decidable fragments:

Local Theory Extensions

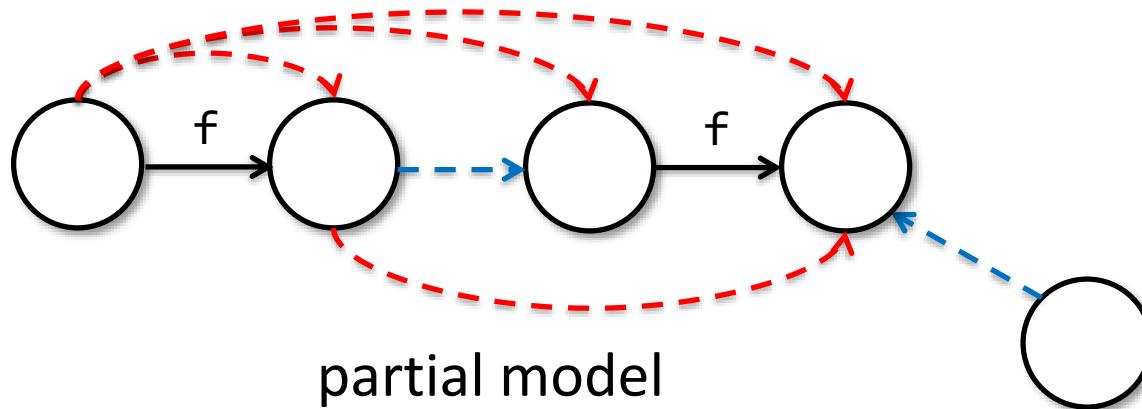
[Sofronie-Stokkermans, CADE'05], [Bansal et al., CAV'15]

Model Completion for Local Theory Extensions



total model

Yields NP decision procedure for GRASS

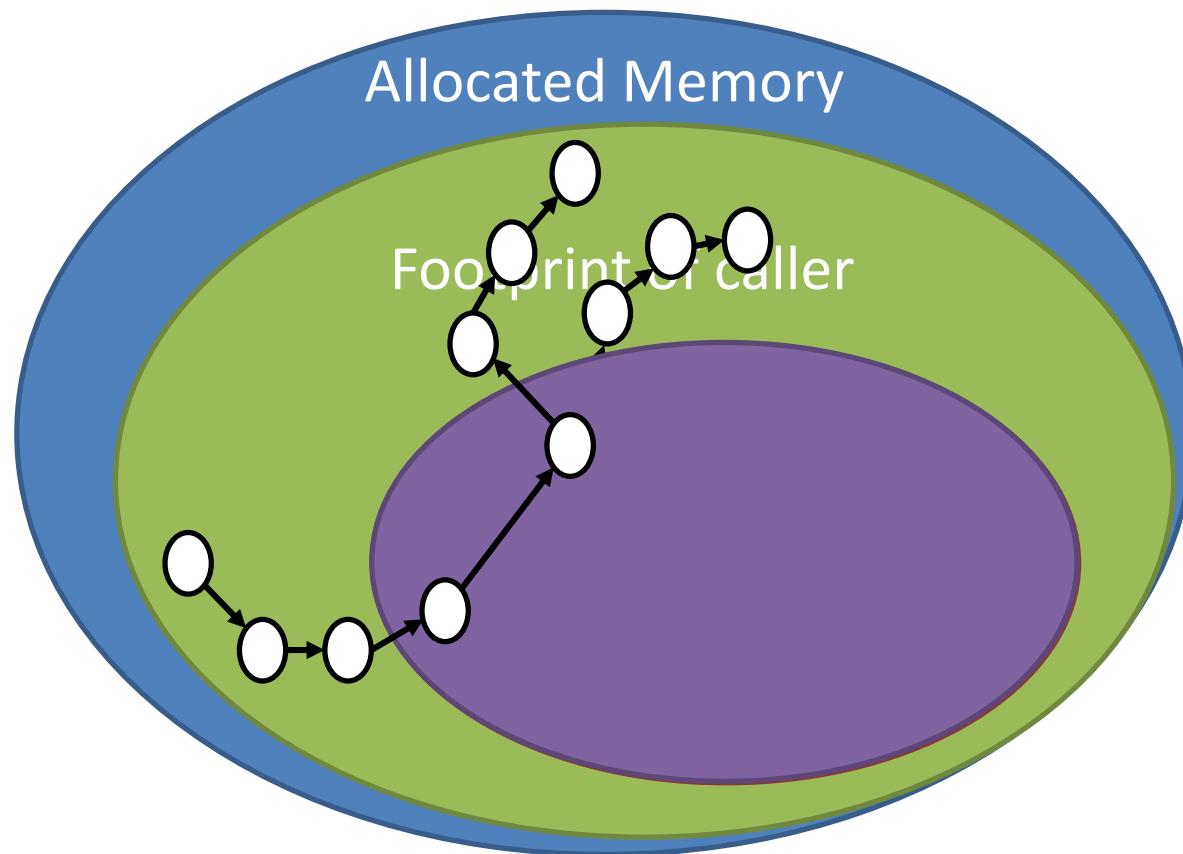


partial model

The Frame Predicate

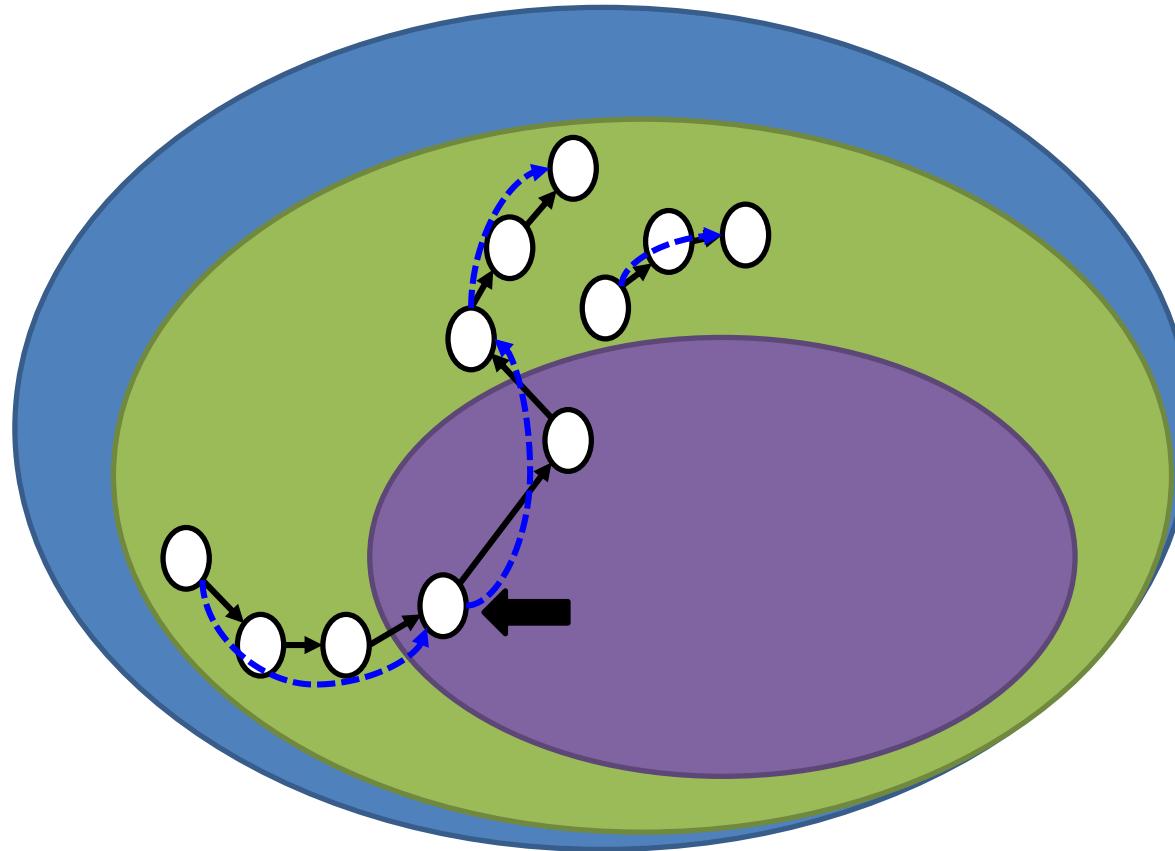
- $\text{Frame}(\text{Alloc}, \text{FP}, \text{next}, \text{next}') \equiv$
 $\forall x. x \in \text{Alloc} \setminus \text{FP} \Rightarrow x.\text{next} == x.\text{next}'$
- Does not work with finite instantiation.
- Need to preserve reachability information in frame.

Axiomatizing the Frame Predicate



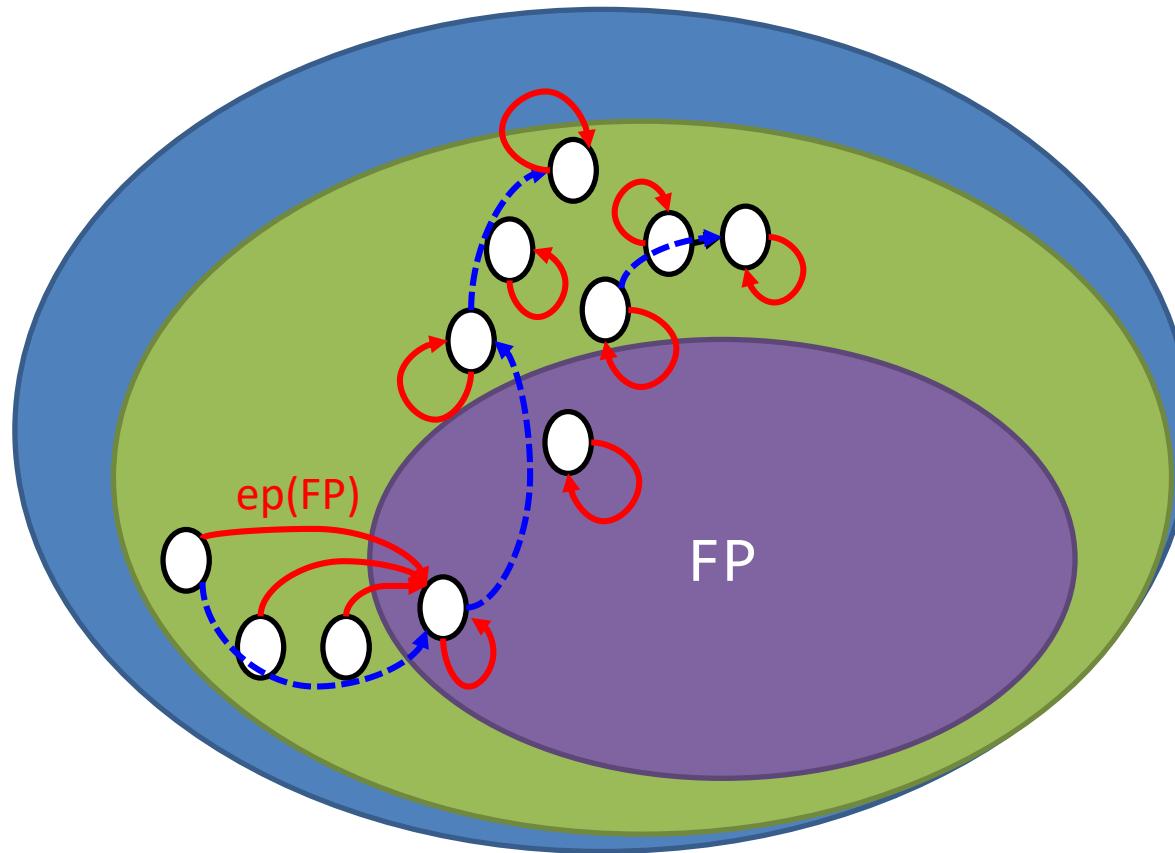
Local changes have global effect on reachability

Axiomatizing the Frame Predicate



Local changes have global effect on reachability
Track **entry points (ep)** into footprint FP

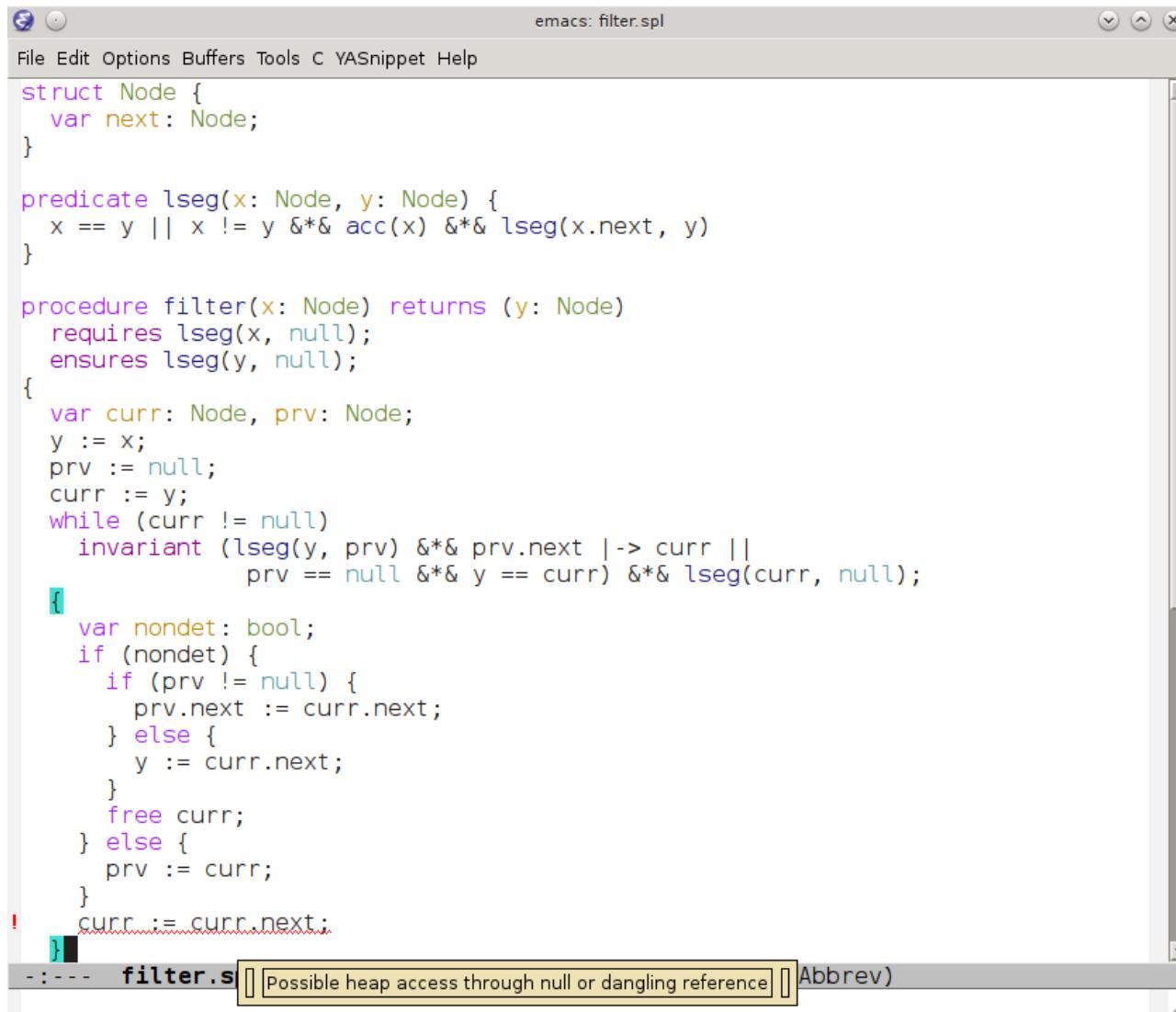
Axiomatizing the Frame Predicate



Local changes have global effect on reachability
Track **entry points (ep)** into footprint **FP**

GRASShopper

<http://github.com/wies/grasshopper>



The screenshot shows an Emacs window titled "emacs: filter.spl". The buffer contains the following SPARK-like code:

```
File Edit Options Buffers Tools C YASnippet Help
struct Node {
    var next: Node;
}

predicate lseg(x: Node, y: Node) {
    x == y || x != y && acc(x) && lseg(x.next, y)
}

procedure filter(x: Node) returns (y: Node)
    requires lseg(x, null);
    ensures lseg(y, null);
{
    var curr: Node, prv: Node;
    y := x;
    prv := null;
    curr := y;
    while (curr != null)
        invariant (lseg(y, prv) && prv.next |-> curr ||
                   prv == null && y == curr) && lseg(curr, null);
    {
        var nondet: bool;
        if (nondet) {
            if (prv != null) {
                prv.next := curr.next;
            } else {
                y := curr.next;
            }
            free curr;
        } else {
            prv := curr;
        }
        curr := curr.next;
    }
}
```

The code defines a struct `Node`, a predicate `lseg`, and a procedure `filter`. The `filter` procedure takes a `Node` `x` and returns a `Node` `y`. It ensures that `y` is the head of a linked list starting from `x`, where every node in the list satisfies the `lseg` predicate. The implementation uses a pointer `curr` to traverse the list and a pointer `prv` to keep track of the previous node. A loop invariant is used to ensure the correctness of the filtering process. The `filter` procedure also includes a `nondet` section and a `free` section.

GRASShopper

<http://github.com/wies/grasshopper>

- Key features
 - C-like language with mixed SL/FOL specifications
 - Compiles to C
 - Supported back-end solvers: Z3 and CVC4
- Benchmarks (several thousand LoC)
 - List data structures
 - Singly/doubly linked, bounded/sorted, with content, ...
 - sorting algorithms, set containers, ...
 - Tree data structures (still in NP!)
 - Binary search trees, skew heaps, union/find, ...
 - Arrays
- Used as backend solver by other tools
 - Viper
 - Starling [Windsor et al. CAV'17]