#### Symbolic Execution and Probabilistic Reasoning

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# Software Safety and Security

- \* Software systems become more pervasive and complex
- Increased need for techniques and tools that ensure safety and security of software systems
- Research directions:
  - automated verification techniques
  - \* application at all phases of software development
  - theoretical foundations and practical tools





## Symbolic Execution

- Systematic program analysis technique King [Comm. ACM 1976], Clarke [IEEE TSE 1976]
- \* Executes programs on symbolic inputs represent multiple concrete inputs
- \* Path conditions conditions on inputs following same program path
  - Check satisfiability using off-the-shelf solvers (Z3) explore only feasible paths
  - Solve path conditions: obtain test inputs
- Bounded execution
- \* Many applications: test-case generation, error detection, ...
- \* Many tools: SAGE, DART, KLEE, Pex, BitBlaze ...
- Symbolic PathFinder

### **Example Concrete Execution**



### **Example Symbolic Execution**



## Loops



## Symbolic PathFinder

- Symbolic execution tool for Java bytecode; opensourced
- \* Lazy initialization for input data structures and arrays
- Handles multi-threading and string operations
- Supports quantitative reasoning
- \* Comes with **library** models



#### https://github.com/SymbolicPathFinder/jpf-symbc

# Test Generation and Bug Finding

#### NASA Applications

- NASA control software: onboard abort executive (OAE) [ISSTA'08]
  - manual testing: time consuming ~ 1 week
  - guided random testing could not obtain full coverage
  - SPF generated ~200 tests to obtain full coverage <1min</li>
  - Flight rules covered 27/27
  - \* Aborts covered 7/7
  - \* Size of input: 27 values/test case
- \* Found major bug in new version

#### OAE structure



# Test Generation and Bug Finding

#### NASA Applications

- Polyglot [ISSTA'11, NFM'12]
  - analysis and test generation for UML, Stateflow and Rhapsody models
  - pluggable semantics for different state chart formalisms
  - analyzed MER arbiter, Ares-Orion communication
  - current work: CoCoSim extensible verification framework for Simulink/ Stateflow
- \* Tactical Separation Assisted Flight Environment (T-SAFE) [NFM'11, ICST'12]
  - integration with Coral heuristic solver for complex mathematical constraints



# Handling Data Structures

 Lazy initialization [TACAS'03,ISSTA'04] — nondeterminism handles aliasing



### Lazy Initialization



- \* collect symbolic constraints **during** concrete executions
- \* DART = Directed Automated Random Testing
- \* Concolic = Concrete / symbolic testing

- \* P. Godefroid, K. Sen and many many others ...
- very popular, simple to implement



![](_page_13_Figure_1.jpeg)

![](_page_14_Figure_1.jpeg)

![](_page_15_Figure_1.jpeg)

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# **Complexity Analysis**

- Problem
  - Estimate the worst-case complexity of programs
- Applications
  - Finding vulnerabilities related to denial-of-service attacks
  - Guiding compiler optimizations
  - Finding and fixing performance bottlenecks in software

![](_page_20_Picture_7.jpeg)

#### DARPA STAC

# Symbolic Complexity Analysis

- Computes inputs that expose worst-case behavior
- Computes bounds on worst-case complexity
- Simple approach
  - \* Perform symbolic execution over the program compute cost of each path
  - \* Return the path with largest cost
  - Scalability issues
- \* Symbolic execution guided by path policies [ICST'17]
  - Encode choices along worst-case path
  - \* Intuition: worst-case behavior for small input can predict worst-case behavior for larger input

## Guided Symbolic Execution

- Policy Generation
  - Exhaustive symbolic execution at small input size(s)
  - Compute path with largest cost
  - Build policy based on decisions taken along that path
- Policy Guided Execution
  - Symbolic execution for increasing input sizes
  - Explore only paths that conform with policy
  - For each input size compute path (and input) with largest cost

![](_page_22_Figure_9.jpeg)

- Function fitting
  - Computes estimate of worst-case behavior as a function of input size
  - \* Gives lower bounds on worst-case complexity for any size

### Path Policies

- \* Decide which branch to execute for the conditions in the program
  - \* Similar to e.g. [Burnim et al. ICSE'09, Zhang et al. ASE'11]
- \* New
  - \* History aware: take into account the history of choices made along a path to decide which branch to execute next
  - \* Context preserving: the decision for each condition depends on the history computed with respect to the enclosing method
- \* Symbolic execution, guided by policies, can reduce to exploring a single path regardless of input size
- \* Scales far beyond non-guided symbolic execution and outperforms previous techniques
- \* **Theoretical guarantee:** when policies are "unified", worst-case path policy is eventually found
  - \* Unification over policies obtained for successive small inputs
  - \* For each condition: take union over decisions specified by each policy

#### Example

```
7 Entry findEntry(String o, ....) {
8   for(Entry e = 1; e!=null; e=e.next) {
9      if (e.key.equals(o)) {
10        return e;
11      }
12   }
....
16   return null;
17 }
```

```
18 class String {
19 char[] value;
20 // ...
    public boolean equals(Object oObj) {
21
22
      // ...
23
      String o = (String) oObj;
24
      if (val.length == o.val.length) {
25
         for(int i=0; i<val.length; i++) {</pre>
26
           if (val[i]!=o.val[i])
27
               return false;
28
29
         return true;
30
      }
31
      return false;
32
    }
33 }
```

![](_page_24_Figure_3.jpeg)

#### Example Application: TextCruncher Sort

- \* Text processing application with various filters, e.g. *WordCount*, *NGramScore*
- Found vulnerability in sorting algorithm
- \* Triggered by files with 3 x n different words: 6000 words: 5 min; 6001 words: few secs.

![](_page_25_Figure_4.jpeg)

## Probabilistic Reasoning

- Extension of symbolic
   execution with probabilistic
   reasoning [ICSE'13,PLDI'14]
  - Computes the probability of a target event, under an input distribution
- Model counting over symbolic constraints
  - Latte, Barvinok -- integer linear constraints, finite domain

![](_page_26_Picture_5.jpeg)

## Probabilistic Reasoning

- \* E.g. assuming uniform distribution,
- Compute path conditions that lead to target event
- Count the number of input values that satisfy the corresponding path conditions
- \* Divide it by the size of the input domain (D)

Probability of event e:  $p(e) = \frac{1}{\#D} \sum_i \#PC_i$   $PC_i \text{ leads to } e.$ 

Example

![](_page_28_Figure_1.jpeg)

Pr(Fail) = #(PC) / #D= #(spinSpeed>70 & discountedPressure >80)/D = 30 x 20/10000 = 6%

# Software Reliability

- Probability of successful termination under stochastic environment assumptions
- Perform bounded symbolic execution: results in three sets of paths
  - \* Success  $PC^{s}$ : lead to successful termination
  - \* Fail  $PC^{f}$ : lead to failure
  - \* Grey  $PC^{g}$ : "don't know"
- \* For given usage profile UP:  $Pr(Fail | UP) = Pr(PC^{t}s | UP)$ , e.g. for uniform UP:  $Pr(Fail) = \#(PC^{t})/D = \#(spinSpeed > 70 \& discounted Pressure > 80)/D = 30 x 20/10000 = 6\%$ .
- \* *Pr(Success | UP)* and *Pr(Grey | UP)* are computed similarly
- \* *Pr(Fail | UP)+Pr(Success | UP)+Pr(Grey | UP)=1*
- \*  $Rel = Pr(Success \mid UP)$
- \* *Confidence* = 1 *Pr*(*Grey* | *UP*) ("1" means that analysis is complete)

## Usage Profiles

![](_page_30_Figure_1.jpeg)

- \* Summarize succinctly hundreds of hours of operation/simulation
- \* UPs can be seen as "pre-conditions"
- Arbitrary UPs handled through discretization
- \* Continuous input distributions [FSE'15]

## Computing with usage profiles

- \* Usage profile: set of pairs <*c*<sub>*i*</sub>, *p*<sub>*i*</sub>>
- ✤ c<sub>i</sub> usage scenario, constraint on inputs
- \*  $p_i$  probability that the input is in  $c_i$

$$Rel = Pr^{s}(P) = \sum_{i} Pr(PC_{i}^{s} \mid UP) =$$
$$= \sum_{i} \sum_{j} Pr(PC_{i}^{s} \mid c_{j}) \cdot p_{j} = \sum_{i} \sum_{j} \frac{\#(PC_{i}^{s} \wedge c_{j})}{\#(c_{j})} \cdot p_{j}$$

# Model Counting

- Latte, Barvinok: integer linear constraints, finite domain —
   Polynomial in number of variables and constraints
  - Omega Lib used for algebraic simplifications
  - Optimizations: independence, caching
- Research on
  - \* model counting for data structures [SPIN'15],
  - \* strings [FSE'16] ABC Solver (UC Santa Barbara)
  - \* non-linear constraints [NFM'17]

## Model Counting for Data Structures

- SPF performs lazy initialization
- Computes Heap PC
- Explicit enumeration using Korat (MIT)
- Complex predicates
  - \* E.g. "acyclic lists of integers with size smaller than the largest contained value"
- Computationally expensive

# Multi-threading

- Enumerate all possible schedules (using model checking, partial order reduction)
  - \* Compute best/worst "reliability"
  - Report best/worst schedule
  - Useful for debugging
- \* Tree-like schedules [ASE'15]
  - \* Monte-Carlo sampling of symbolic paths
  - Reinforcement learning used to iteratively compute schedules

### Application: Onboard Abort Executive

- NASA control software
  - Mission aborts
  - \* 3754 paths, 36 input sensors
  - \* 30 usage scenarios
  - Execution time: 20.5 sec
  - \* Checking for "no aborts"
  - \* Rel > 0.9999999

![](_page_35_Picture_8.jpeg)

## Side-Channel Analysis

#### Side-channel attacks

- recover secret inputs to programs from non-functional characteristics of computations
- time or power consumption, number of memory accesses or size of output files

An attack on "main" channel: exponential On "side channel": linear

![](_page_36_Picture_6.jpeg)

## Side-Channel Analysis

- Non-interference too strict
- \* Quantitative Information-Flow Analysis (QIF) to determine information leakage
- \* Perform symbolic execution (high and low symbolic)
- \* Collect all symbolic paths each path leads to an observable
- \* Side channels produce a set of "observables" that partition the secret
- \* *Cost model* for observables: execution time, number of packets sent/received over network, etc.

 $\mathcal{O} = \{o_1, o_2, ... o_m\},\$ 

#### Quantifying Information Leakage

**Channel Capacity** 

$$CC(P) = log_2(|\mathcal{O}|)$$

Shannon Entropy

$$\mathcal{H}(P) = -\sum_{i=1,m} p(o_i) \log_2(p(o_i))$$

## Computing Shannon Entropy

$$\mathcal{H}(P) = -\sum_{i=1,m} p(o_i) \log_2(p(o_i))$$

Use symbolic execution and model counting

the probability of observing 
$$o_i$$
 is:  

$$p(o_i) = \frac{\sum_{cost(\pi_j)=o_i} \#(PC_j(h, l))}{\#D}$$

### Password Example

```
// 4-bit input and password; D=256
boolean verifyPassword(byte [] input,
                            byte [] password){
   for(int i = 0; i < SIZE; i++){
     if (password[i]!=input[i])
        return false ;
   Thread.sleep(25L);
   }
   return true;
}</pre>
```

// 4-bit input and password; D=256
boolean verifyPassword(byte [] input,
 byte [] password){
 boolean matched=true;
 for(int i = 0; i < SIZE; i++){
 if (password[i]!=input[i])
 matched=false ;
 else
 matched=matched;
 Thread.sleep(25L);
 } return matched; }</pre>

\* 5 paths

- \* *h*[0]!=*l*[0] returns false: 128 values
- *h*[0]=*l*[0] & *h*[1]!=*l*[1] returns false:
   64 values
- *h*[0]=*l*[0] & *h*[1]=*l*[1] & *h*[2]!=*l*[2]
   returns false: 32 values
- *h*[0]=*l*[0] & *h*[1]=*l*[1] & *h*[2]=*l*[2] &
   *h*[3]!=*l*[3] returns false: 16 values
- h[0]=l[0] & h[1]=l[1] & h[2]=l[2] & h[3]=l[3] returns true: 16 values

Observable is time: *H*=1.875 Observable is output: *H*=0.33729

## Maximizing Leakage

```
void example(int lo, int hi) {
  if(lo<0) {
    if(hi<0) cost=1;
    else if(hi<5) cost=2;
    else cost=3;
  }
  else {
    if(hi>1) cost=4;
    else cost=5;
  }
}
```

- using symbolic low value overapproximates leakage
- \* example: 5 possible observables; lo<0:</li>
  3 observables, lo≥0: 2 observables

- Goal: find low input that maximizes number of observables (channel capacity)
- Shows most powerful "attack" in one step
- Shows most vulnerable program behavior

## Maximizing Leakage using MaxSMT

```
void example(int lo, int hi) {
  if(lo<0) {
    if(hi<0) cost=1;
    else if(hi<5) cost=2;
    else cost=3;
  }
  else {
    if(hi>1) cost=4;
    else cost=5;
  }
}
```

```
C_{1} :: (l < 0 \land h_{1} < 0)
C_{2} :: (l < 0 \land h_{2} \ge 0 \land h_{2} < 5)
C_{3} :: (l < 0 \land h_{3} \ge 5)
C_{4} :: (l \ge 0 \land h_{4} > 1)
C_{5} :: (l \ge 0 \land h_{5} \le 1)
```

MaxSMT solution: Lo=-1 satisfies first 3 clauses Leakage  $\log_2(3)=1.58$  bits

 MaxSMT solving — generalization of SMT to optimization

- given a set of weighted clauses
- find solution that maximizes the sum of the weights of the satisfied clauses
- Assemble PCs that lead to same observable into "clauses" of weight "1"
- MaxSMT solution gives maximal assignment ⇒ largest number of observables
- Any other assignments lead to fewer observables

## Multi-run Analysis

- \* The attacker learns the secret by observing multiple program runs
- Generalization to multiple-run side-channel analysis

 $P(h, l_1); P(h, l_2); ...P(h, l_k)$ 

- \* An "observable" is a **sequence** of costs
- MaxSMT used to synthesize a sequence of public inputs that maximize leakage; nonadaptive attacks; greedy approach [CSF'16]
- Maximize Shannon leakage: parameterized model counting+ numerical optimization; adaptive attacks [CSF'17]
- Analysis of password examples and cryptographic functions
- \* Shown experimentally to perform better than previous approaches based on self composition or brute-force enumeration
- \* More work on side-channel analysis [ISSTA'18]

#### **Results for Password Check**

#### Results for 4 elements with 4 values (8 bits of information)

![](_page_43_Figure_2.jpeg)

Timing Side Channel

# Current/Future Work

#### Monte Carlo Tree Search For SW Analysis

- \* Monte Carlo Tree Search [SEFM'18]
  - \* Heuristic search algorithm; Iterative expansion of search tree to find optimal decisions
  - \* State-of-the art results in solving Go, board games, poker
  - \* Good for domains modeled as a tree
- \* Sampling along symbolic paths for increased scalability
  - \* Symbolic paths represent multiple concrete paths; Organized in tree
  - Optimize with respect to the longest path (highest reward)
- \* Aggressive pruning of state space
  - \* Speeds up analysis and guarantees convergence

![](_page_45_Figure_10.jpeg)

# Symbolic Execution and Fuzzing

- \* Fuzzing: random testing with some guidance
  - \* cheap
  - not good at finding "deep paths" that depend on complicated constraints
- Symbolic execution
  - expensive
  - good at finding deep paths
- \* Better together!
- \* See **Badger** talk at ISSTA'18 on Wednesday

#### Probabilistic Analysis for Autonomous Vehicles

#### SafeTugs

- Currently aircraft either needs to use their engines or be towed during departure/arrival ground operations
- \* Engines off is more efficient
- \* This project will focus on autonomous tugs for towing

#### Analysis

- Predictive analysis for safe surface and air operation [HLDVT'16]
  - \* Involves model inference from telemetry/simulation data
- "Simulation" environment using Symbolic PathFinder and probabilistic reasoning [PHS'16]
  - Planning phase generates a plan of tug movement on a grid (abstraction of the airport)
  - The plan is given as input to SPF; calculate how robust the plan is when the probabilities are changed
  - The output of our tool can be used to trigger dynamic re-planning during operation

Progress: see workshop talk on Thursday at TAV-CPS/IoT !

![](_page_47_Picture_13.jpeg)

#### Checking Robustness of Deep Neural Nets

- \* Deep Learning
  - \* Machine learning that enables representation and modeling of complex non-linear relationships
  - Neural Networks (feed-forward, convolutional), Deep Belief Networks
- Application domains:
  - Pattern analysis, image classification, speech/audio recognition, perception modules in self-driving cars
  - High-dimensional, Classifiers are non-linear and potentially discontinuous
- \* Deep Neural networks are vulnerable to adversarial inputs:
  - siven input x, find new input x' that is "similar" to x but is assigned a class different from x by the network [Szegedy et. al. 2013]?
- \* Current research: use symbolic execution and k-means clustering for robustness check
  - \*  $|x-x'| < d \Rightarrow F(x) = F(x')$ ; a counterexample is an adversarial input
  - \* w/D. Gopinath (CMU)
  - \* see ATVA'18 talk!

![](_page_48_Picture_13.jpeg)

Street sign

![](_page_48_Picture_15.jpeg)

Birdhouse

![](_page_48_Picture_17.jpeg)

ship

truck

# Quantification of Software Changes

- \* Programs evolve during development and maintenance
  - \* There is a need for detection and characterization of software changes
- Current techniques
  - Syntactic: diff, imprecise, leads to unnecessary maintenance work
  - Behavioral: check logical implication between behavioral abstractions: yes/no answers
- Quantitative representation of program change [ASE'15]
- Probability of reaching program events how that evolves in time
  - \* rank program versions based on probability of failure
  - \* after bug fixing probability of failure should decrease
- Percentage of inputs affected by change
  - measurable delta between program versions
  - measurable effort to re-test
- Automated program repair:
  - \* rank repairs based on probability of success/failure

![](_page_49_Picture_15.jpeg)

### Conclusion

- Symbolic execution and its extension to probabilistic reasoning
- Applications in program analysis for safety and security
- Future directions ...
  - \* Leakage computation for noisy side channels [CSF'18]
  - Distributed analysis— lots of opportunities for symbolic execution
  - Combinations with fuzzing
- Challenges
  - scalability; handling loops
  - non-linear numeric constraints, string constraints: constraint solving, (parametrized) model counting, MaxSMT

![](_page_51_Picture_0.jpeg)

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