The Maude-NRL Protocol Analyzer Lecture 2: Controlling the Search Space and Asymmetric Unification

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Outline

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1. Controlling the Search Space
   - Learn-Only-Once and Grammars
   - Other Ways of Reducing the Search Space
2. Asymmetric Unification
   - Background and Motivation
   - A New Unification Paradigm: Asymmetric Unification

How Maude-NPA Controls the Search Space

- Left to itself, Maude-NPA will search forever
- Must use techniques for ruling out redundant or provably unreachable states to obtain finite search space
- We have developed a number of different techniques for doing this:
  - Intruder learns only once
  - Grammars
  - Subsumption
  - Super-Lazy Intruder
- We will cover these in this lecture

Important Assumptions

- Equational theory is of the form $E = R \uplus \Delta$
- R is a set of rewrite rules and Δ is regular
- In any states produced by Maude-NPA $t{\in}\mathcal{I}$, $t{\notin}\mathcal{I}$, and negative terms are R-irreducible
- Furthermore, no substitutions produced by further search will make these terms reducible
    - Reason: many of the checks made by Maude-NPA for state space reduction rely on the presence of particular sub terms
    - Allowing these sub terms to vanish because of rewrite rules or further substitutions will invalidate the checks
    - We will show how these assumptions are guaranteed in the lecture on the asymmetric unification

Outline

Two basic restrictions of the search space

Powerful tools:

1. Learn-only-once: any terms the intruder will learn in the future can't already be known
2. Grammars describing unreachable states: the intruder learns a term in the language described by the grammar only if he/she knew another term in the language in a past state

Motivating Example

- Consider protocol with:
  - Two operators
    - $e(K, X)$ stands for encryption of message X with key K
    - $d(K, X)$ stands for decryption of message X with key K
  - Two regular strands: (Dolev-Yao):
    - $[-(X), +(d(k, X))]$
    - $[+(e(k, r))]$

    Two Intruder strands
    - $[-(K), -(X), +(d(K, X))]$
    - $[-(K), -(X), +(e(K, X))]$
  - One equation
    - $d(K, e(K, X)) = X$

A Partial (Backwards) Search Tree

$$
\begin{tikzcd}
 & t \arrow[d] & & \\
 & \{e(k,t)\} \arrow[dl] \arrow[dr] & & \\
\{k,t\} \arrow[d] & & \{e(k,e(k,t))\} \arrow[dl] \arrow[dr] & \\
\text{stop} & \{e(k,t),k\} \arrow[d] & & \{e(k,e(k,e(k,t)))\} \arrow[d] \\
 & \text{stop} & & \cdots
\end{tikzcd}
$$

Powerful tools:

(1) Learn-only-once: terms the intruder will learn in the future and doesn't know in he past.

(2) Unreachable states: the intruder learns a term in a family only if he/she knew another term in that family in a past state

(1) Learn-Only-Once Restriction

- Suppose in looking for a term t, you find a state where the intruder knows the same t, then cut the search space

$$
\begin{tikzcd}
& t \arrow[d] \\
& \{e(k,t)\} \arrow[dl] \\
\{k,t\} \arrow[d] & \\
\text{stop} &
\end{tikzcd}
$$

- Can tell if intruder has not learned X by seeing if intruder will learn X in the future

(2) Grammars characterizing unreachable states

$$
\begin{array}{lll}
Z \npreceq r & & \\
\downarrow & & \\
\{e(K,Z)\} & & \\
\quad\searrow & & \\
 & \{e(K,e(K,Z))\} & \\
 & \quad\searrow & \\
 & & \{e(K,e(K,e(K,Z)))\} \\
 & & \downarrow \\
 & & \ldots
\end{array}
$$

- Discover Grammars providing infinite set of terms intruder can't learn.
  1. $t \in L$
  2. $Z \in L \mapsto e(Y,Z) \in L$

  1. $Z \notin \mathcal{I},\ Z \npreceq r \mapsto e(A,Z) \in L$ ($Z \npreceq r$ means Z not subsumed by r)
  2. $Z \in L \mapsto e(Y,Z) \in L$
- If the intruder learns a term in the language, then he/she must have learned another term in a state in the past.

Grammar Generation Is Automated

- Start with initial grammar, giving a single term known by the intruder, along with conditions on the term, such as some sub term not yet known by the intruder
  - Maude-NPA uses function symbol definitions in protocol spec as source for initial grammars
  - User can define own initial grammars if desired, either in addition to or in place of Maude-NPA grammars
- Maude-NPA finds the terms the intruder needed to know to generate these terms
- Checks if new terms are also in the language defined by the grammar
- If not, uses a set of heuristics to add new grammar rules
- If no heuristic applies, adds an exception to the grammar rule
- Repeats this process until it reaches a fixed point
- In cases Maude-NPA fails to generate a grammar, it provides the reasons for its failure

Status of Grammars

- Grammar generation heuristics little changed from original NRL Protocol Analyzer
- Works well on most theories we've tried
- Main exceptions are exclusive-or and Abelian groups: presence of inverses causes unexpected behavior
    - Grammar generation heuristics rely on assumptions about term on LHS of grammar rule being sub term of RHS of grammar rule
    - Not satisfied by grammars produced by these theories
    - Have a partial work-around for exclusive-or
- Currently planning to rethink grammars in order to address this

Outline

Other Ways of Reducing Search Space

- Grammars can reduce infinite to finite, but may still need to cut search space size for efficiency purposes
  - In some cases, grammars alone not enough to reduce infinite to finite, and we need other techniques as well
- We have developed a number of different techniques, and we describe them now
  - Execute Rule 1 First
  - Subsumption Partial Order Reduction
  - Use Power of Strands to See Into Past and Future
  - Super-Lazy Intruder

Execute Rule 1 First

- If there is a strand of the form $[\ l_1, u^- \mid l_2\ ]$ present, execute the rule replacing it by $[\ l_1 \mid u^-, l_2\ ]\ , u \in \mathcal{I}$ first
- If there are several fix an order and execute them all first, in that order
- Removes extra step introduced by converting negative terms to intruder terms
- Implementing this doubled the speed of the tool
    - Not surprising, because replaced two steps by one

Subsumption Partial Order Reduction

- Partial order reduction standard idea in model checking, used in a lot of protocol analysis tools, too
  - Identify when reachability of state S_1 implies reachability of S_2 and remove S_1
  - In Maude-NPA, this happens, roughly, when $S_2 \subseteq S =_B \sigma S_1$ for some substitution σ
  - Can then eliminate S_1

Using the Power of Strands

- Strands allow you to see the past and the future of a local execution
- Helpful since Maude-NPA very sensitive to the past and future
- Things we've done so far
  - If a term $x \notin \mathcal{I}$ and a strand $[\ l_1, -(x), l_2 \mid l_3\ ]$ both appear in a state, then the state is unreachable
    - Reaching it would require violation of intruder-learns-once
  - Let f and g be two terms containing $n(A, r)$. If
    - $f \in \mathcal{I}$ appears in a state, and;
    - $[\ l_1 \mid l_2, +(g), l_3,\ ]$ also appears, with strand identifier containing r and no $n(A, r)$ term in l_1;

    Then reaching the state requires the intruder to learn a nonce before it is generated and thus is unreachable.

Super-Lazy Intruder

- Based on an idea of David Basin, plus a trick used by the old NPA
- If a term $X \in \mathcal{I}$ appears in a state, where X is a variable, we assume that the intruder can easily find x, and so safe to drop it
- Super-lazy intruder: drop terms made out of variable terms, e.g. X;Y and e(K,Y)
- Need to revive variable terms if they later become instantiated
- Solution: keep the term, and state it appears in, around as a "ghost"
  - Revive the ghost, replacing current state by ghost term and ghost state, but with current substitutions to variables if any variable subterm becomes instantiated

Experimental Results 1

Protocol	none					Grammars					%
NSPK	5	19	136	642	4021	4	12	49	185	758	81
NSL	5	19	136	642	4019	4	12	50	190	804	79
SecReT06	1	6	22	119	346	1	2	6	15	36	89
SecReT07	6	20	140	635	4854	6	17	111	493	3823	21
DH	1	14	38	151	816	1	6	14	37	105	87

Protocol	none					Input First					%
NSPK	5	19	136	642	4021	11	123	1669	26432	N/A	0
NSL	5	19	136	642	4019	11	123	1666	26291	N/A	0
SecReT06	1	6	22	119	346	11	133	1977	32098	N/A	0
SecReT07	6	20	140	635	4854	11	127	3402	N/A	N/A	0
DH	1	14	38	151	816	14	135	1991	44157	N/A	0

Protocol	none					Inconsistency					%
NSPK	5	19	136	642	4021	5	18	95	310	650	83
NSL	5	19	136	642	4019	5	18	95	310	650	83
SecReT06	1	6	22	119	346	1	6	22	114	326	5
SecReT07	6	20	140	635	4854	6	18	107	439	3335	31
DH	1	14	38	151	816	1	12	12	56	128	84

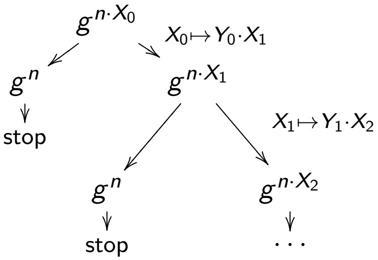
Experimental Results 2

Protocol	none					Transition Subsumption					%
NSPK	5	19	136	642	4021	5	15	61	107	237	94
NSL	5	19	136	642	4019	5	15	61	107	237	94
SecReT06	1	6	22	119	346	1	6	15	39	78	77
SecReT07	6	20	140	635	4854	6	15	61	165	506	89
DH	1	14	38	151	816	1	14	26	102	291	64

Protocol	none					Super-lazy Intruder					%
NSPK	5	19	136	642	4021	5	19	136	641	3951	1
NSL	5	19	136	642	4019	5	19	136	641	3949	2
SecReT06	1	6	22	119	346	1	6	22	119	340	2
SecReT07	6	20	140	635	4854	6	16	44	134	424	91
DH	1	14	38	151	816	1	14	38	138	525	35

Protocol	none					All optimizations					%
NSPK	5	19	136	642	4021	4	6	4	2	1	99
NSL	5	19	136	642	4019	4	7	6	2	0	99
SecReT06	1	6	22	119	346	2	3	2	-	-	99
SecReT07	6	20	140	635	4854	5	1	1	1	-	99
DH	1	14	38	151	816	4	6	10	9	12	99

Infinite Behavior We Aren't Able to Prevent



- Different from rewrite-rule based grammar behavior, because infinite behavior results from substitution
- Root term grows larger instead of leaf terms
- Behavior becomes more common as theories grow more complex
- Currently we can just cut off branch after a certain point
- But, would like a method that guarantees completeness

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The Problem

- Protocol analysis tools often depend on syntactic properties of terms that fail to be invariant under equational theories
- Check for nonces appearing as subterms
  - Logical systems: CPSA, PCL, PDL, to determine which actions should precede others
  - Maude-NPA: to rule out unreachable states
- Depth of terms
  - ProVerif : option to ensure termination
- Syntactic pattern matching
  - Maude-NPA : to rule out infinite search paths
- In this lecture I'll describe how we're dealing with this problem in Maude-NPA, and how we think our techniques could be applied to other unification tools

An Example

- Start with exclusive-or $\oplus$
  - $\oplus$ is AC, with additional equations $x \oplus 0 = x$ and $x \oplus x = 0$.
- Consider the following protocol
  1. $A \to B : pke(B, N_A)$
  2. $B \to A : N_B \oplus N_A$
- A checks that the message she receives is $Z \oplus N_A$ for some Z
  - How it works in Maude-NPA
  - `::r::[nil, +(pke(B,n(A,r))),-(Z [+] n(A,r)), nil ]`
  - Unify `Z [+] n(A,r) with some term in the intruder's knowledge`
- So, what if $Z = Y \oplus N_A$?

How we handle this in Maude-NPA

- Express equational theory as $R = \{X \oplus 0 \to X, X \oplus X \to 0, X \oplus X \oplus X \to X\} \uplus \Delta = \mathrm{AC}$
    - nonce containment invariant under AC
    - R is a set of rewrite rules convergent and terminating wrt AC
- Find all the possible reduced forms of `Z [+] n(A,r)` wrt R
    - There are two:
        - `< Z [+] n(A,r), id >`
        - `< Y, Z / Y [+] n(A,r) >`
- One strand for each reduced form
    - `::r::[nil, +(pke(B,n(A,r))),-(Z [+] n(A,r)), nil ]`
    - `::r::[nil, +(pke(B,n(A,r))),-(Y), nil ]`
- Include constraints that negative terms in strands are irreducible wrt R
- Any further substitution made in the search process must obey these constraints

Three Things we need to make this work

1. Characterize theories in which every term has a finite number of reduced forms
   - We understand this: this is equivalent to the finite variant property
2. Unification algorithms giving a set of mgu's Σ $x =?y$ such that for all $\sigma \in \Sigma$, σy is irreducible
   - We call this *asymmetric unification*
   - *Variant narrowing* has this property, we are looking for more efficient algorithms
3. Combine these into a sound and complete search strategy

Using Variant Narrowing to Satisfy Irreducibility Constraints in Protocol Analysis

- Recap from previous lecture
- For each strand St in a specification, compute a most general set of variants V of the the negative terms
- For each variant (σ, t) create a new strand σSt
- Each time a positive term s is unified what a term t in the intruder knowledge, user asymmetric unification to find a complete set of unifiers of s and t that leave t irreducible, as well as all negative terms already present in the state

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2. Asymmetric Unification
   - Background and Motivation
   - A New Unification Paradigm: Asymmetric Unification

Asymmetric Unification

- Let $(\Sigma, R \uplus \Delta)$ be a an equational theory, where R is a set of rewrite rules confluent, terminating and coherent wrt Δ.
- A solution to an asymmetric unification problem $s_1 =_{\downarrow} t_1 \wedge \ldots \wedge s_k =_{\downarrow} t_k$ is a substitution σ such that
  1. For each i, $\sigma s_i =_{\Delta} \sigma t_i$
  2. For each i, σt_i is irreducible
- A set Θ is a most general set of asymmetric unifiers of P if for any asymmetric unifier there σ there is a $\theta \in \Theta$ such that $\sigma =_{\Delta} \tau\theta$ for some τ.

Some Examples from XOR

1. $c =?X \oplus Y$, S-unifiable, but not A-unifiable
2. $a + b =?X \oplus Y$
   - $\sigma = [X \mapsto Y \oplus a \oplus b]$ is a most general S-unifier, but not an A-unifier
   - $[X \mapsto a, Y \mapsto b], [X \mapsto b, Y \mapsto a]$ is a set of most general A-unifiers
3. $X =?Y \oplus X$
   - $[Y \mapsto X \oplus Z]$ an a most general S-unifier but not an A-unifier
   - $[X \mapsto Y \oplus Z]$ is equivalent, but is an A-unifier
4. $Z =?X_1 \oplus X_2, Z =?Y_1 \oplus Y_2$
   - $\sigma = [X_1 \mapsto Z \oplus X_2, Y_1 \mapsto X \oplus Y_2]$ is a most-general S-unifier, but not an A-unifier
   - $\sigma = [Z \mapsto X_1 \oplus V \oplus Y_2, X_2 \oplus V \oplus Y_2, Y_1 \oplus X_1 \oplus V]$ is equivalent and an A-unifier

Using Variant Narrowing to Find Most General Set of Asymmetric Unifiers

- Let $(\Sigma, R \uplus \Delta)$ be a an equational theory, where R is a set of rewrite rules confluent, terminating and coherent wrt Δ.
- Furthermore, assume that $(R \uplus \Delta)$ has the finite variant property
- Given a problem $s =_{\downarrow} t$
  1. Use variant narrowing to find a set of most general variants V of s.
  2. Discard any $(\sigma, \sigma s \downarrow) \in V$ such that σt is reducible
  3. For each remaining $(\sigma, \sigma s \downarrow)$ find set of mgu's of $\sigma s \downarrow =_E ? \sigma t$.
  4. Discard any unifier θ such that $\theta \sigma t$ is reducible
  5. Remaining set is a set of most general asymmetric unifiers
- Can we do better (e.g. faster)?

Next Step: Asymmetric Unification (AU) as a Problem in its Own Right

- As far as we can tell, no-one has studied this before
- Narrowing only algorithm we know of that can achieve this
- What we do have found so far
  - AU at least as hard as symmetric unification (SU)
    - Any SU problem $s =?t$ can be turned into AU problem $s =?X, t =?X$.
  - AU *strictly harder* than SU - XOR without any other symbols is in P for SU but NP-complete for AS
  - SU can be unitary while AU is not (XOR)
  - There exist theories for which SU is decidable but AU is not
- We are working on a general approach for converting equational unification algorithms to asymmetric unification algorithms
- Have applied it to unification in XOR theory (ACU + cancellation)

Outline for a General Procedure

- Start with a decomposition $R \uplus Delta$ and a unification algorithm
- GIven a problem $x =?y$, find a complete set of unifiers Σ using the symmetric algorithm
- For each $\sigma \in \Sigma$
  1. If σ is an A-unifier, keep it
  2. If not, see if there is an equivalent A-unifier σ' and if so, replace σ with σ'
  3. If not, apply 1) and 2) to more completely instantiated versions σ and replace σ' with those
  4. If none of those work, discard σ
- Application to exclusive-or given in Ertabur et al. 2012 and 2013.
- Papers also includes experimental results

Experimental Results: Unification

Unif. Problem	T. A-V	# A-V	T. D-A	# D-A	% T.	% #
$NS_1 \oplus NS_2 =_{\downarrow} NS_3 \oplus N_A$	153	12	153	1	0	91
$NS_1 \oplus N_A =_{\downarrow} NS_2 \oplus NS_3$	137	5	121	1	11	80
$NS_1 \oplus NS_2 =_{\downarrow} NS_3 \oplus NS_4 \oplus NS_5$	286	54	116	1	59	98
$NS_1 \oplus NS_2 =_{\downarrow} NS_3 \oplus NS_4 \oplus N_A$	159	36	115	1	27	97
$NS_1 \oplus NS_2 =_{\downarrow} N_A$	127	4	114	1	10	75
$NS_1 \oplus NS_2 =_{\downarrow} null$	128	1	105	1	17	0
$NS_1 \oplus NS_2 =_{\downarrow} null \oplus NS_3$	130	7	105	1	20	85
$M_1 \oplus M_2 =_{\downarrow} M_3 \oplus pair(V_1, M_4)$	51	12	44	1	13	91
$pair(V, rc4(V_1, kAB) \oplus ([N_A, c(N_A)]))$ $=_{\downarrow} pair(V_1, M_1)$	30	1	29	1	3	0
$M_1 \oplus M_2 =_{\downarrow} M_3 \oplus V_1$	33	12	32	1	3	91
$M_1 \oplus M_2 =_{\downarrow} M_3 \oplus ([N_1, c(N_2)])$	34	12	30	1	11	91
$M_1 \oplus M_2 =_{\downarrow} M_3 \oplus pair(V_1, pair(V_2, M_4))$	36	12	30	1	16	91

- T. A-V , # A-V = time and number of unifiers, resp. of variant unification
- T. D-A, # D-A = time and number of unifiers, resp. of asymmetric algorithm
- % T. and % # = improvement represented as percentage of A-V score

Experimental Results: Protocol Analysis

states/seconds	1 step	2 steps	3 steps	4 steps	5 steps
RP - Standard	2/0.08	5/0.16	13/0.86	49/3.09	267/17.41
RP - Asymmetric	1/0.03	45/1.08	114/2.26	1175/37.25	13906/4144.30
WEPP - Standard	5/0.09	9/0.42	26/1.27	106/5.80	503/ 34.76
WEPP - Asymmetric	4/0.05	9/0.12	26/0.64	257/144.65	2454/612.08
TMN - Standard	5/0.11	15/ 0.55	99/3.82	469/ 25.68	timeout
TMN - Asymmetric	4/0.06	24/0.53	174/3.63	1079/170.29	9737/1372.55

- Protocol analysis experiments with regular XOR unification algorithm vs using asymmetric XOR unification algorithm.
- A pair n/t means: $n =$ number of states, and $t =$ time in seconds

Asymmetric Unification Over Combinations of Theories : Strategy 1

- Suppose that $E_1 = (R_1 \uplus \Delta_1)$ and $E_2 = (R_2 \uplus \Delta_2)$ have asymmetric unification algorithms $\mathcal{A}_\infty, \mathcal{A}_\in$
- How do we find an asymmetric algorithm $\mathcal{A}$ for $E = ((R_1 \cup R_2) \uplus (\Delta_1 \cup \Delta_2))$?
- $(R_1 \uplus \Delta_1)$ and $(R_2 \uplus \Delta_2)$ are both finite variant decompositions, and a unification algorithm exists for $\Delta_1 \cup \Delta_2$
  - If $((R_1 \cup R_2) \uplus (\Delta_1 \Delta_2))$ is FVP, then can use variant narrowing
  - Although this is not decidable, tools and semi decision procedures exist

Asymmetric Unification Over Combinations of Theories: Strategy 2

- If both $\mathcal{A}_\infty$ and $\mathcal{A}_\in$ satisfy a condition known as *linear constant restriction* than a general combination procedure exists (Erbatur et al., 2014), based on Baader-Schwarz method (1996)
  - Linear constant restriction basically means algorithm still works with additional free constants added to theory
  - Needed for Baader-Schwarz result to
  - $\mathcal{A}$ highly nondeterministic, but may be able to adapt known optimization techniques
- Both strategies result in inefficient algorithms: can we do better?

Conclusion

- We've described a way of decomposing and attacking unification problems that we originally adopted for convenience in Maude-NPA
- We found that it also makes it much easier to apply state space reduction techniques that rely upon syntactic checking
- We believe that this has applications, not only to Maude-NPA, but to other tools and approaches as well
- With that in mind, we are starting to investigate this approach in a more general and systematic way

Asymmetric Unification References

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