Programming for everyone: from solvers to solver-aided languages and beyond

Emina Torlak

U.C. Berkeley

emina@eecs.berkeley.edu

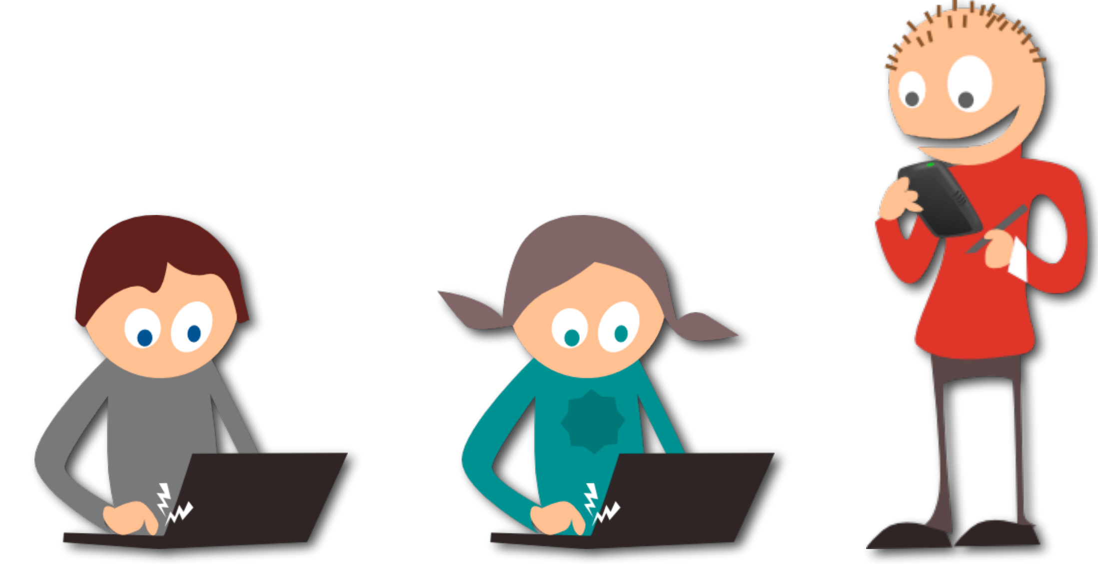
http://people.csail.mit.edu/emina/

vision

a little programming for everyone

A little programming for **everyone**

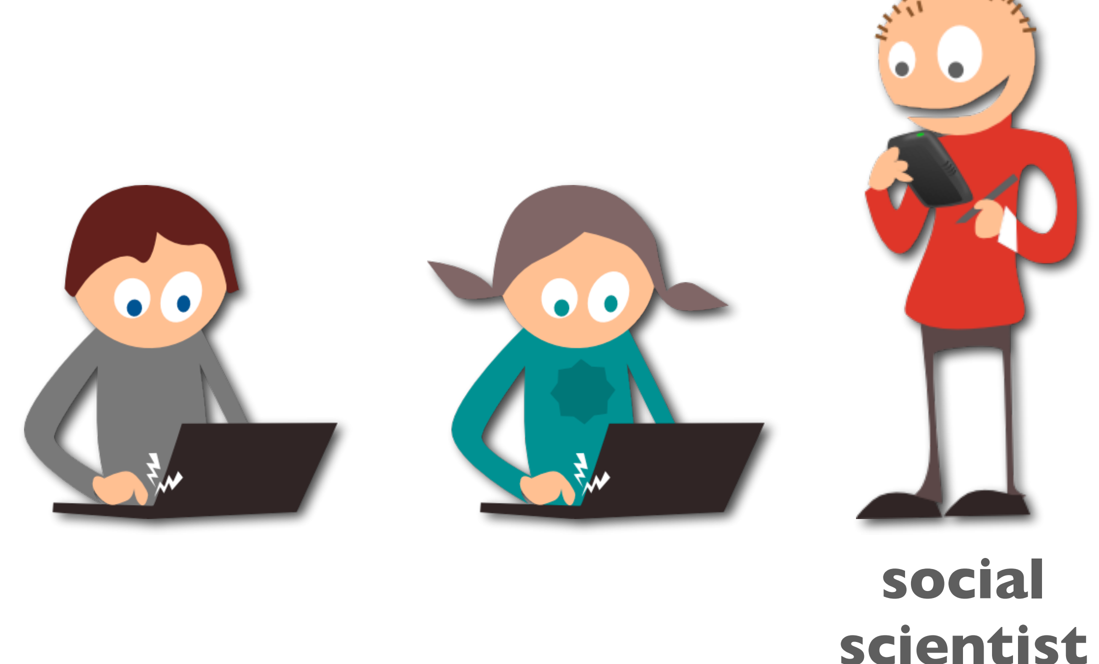
We all want to build programs …



A little programming for everyone

We all want to build programs …

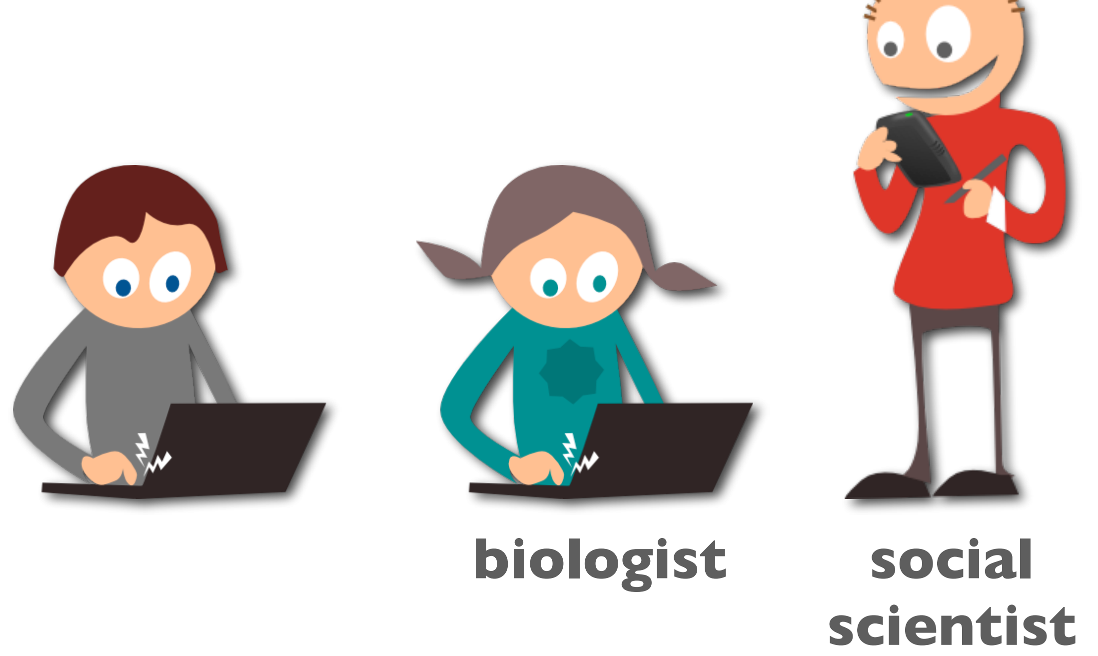
- spreadsheet data manipulation



A little programming for **everyone**

We all want to build programs …

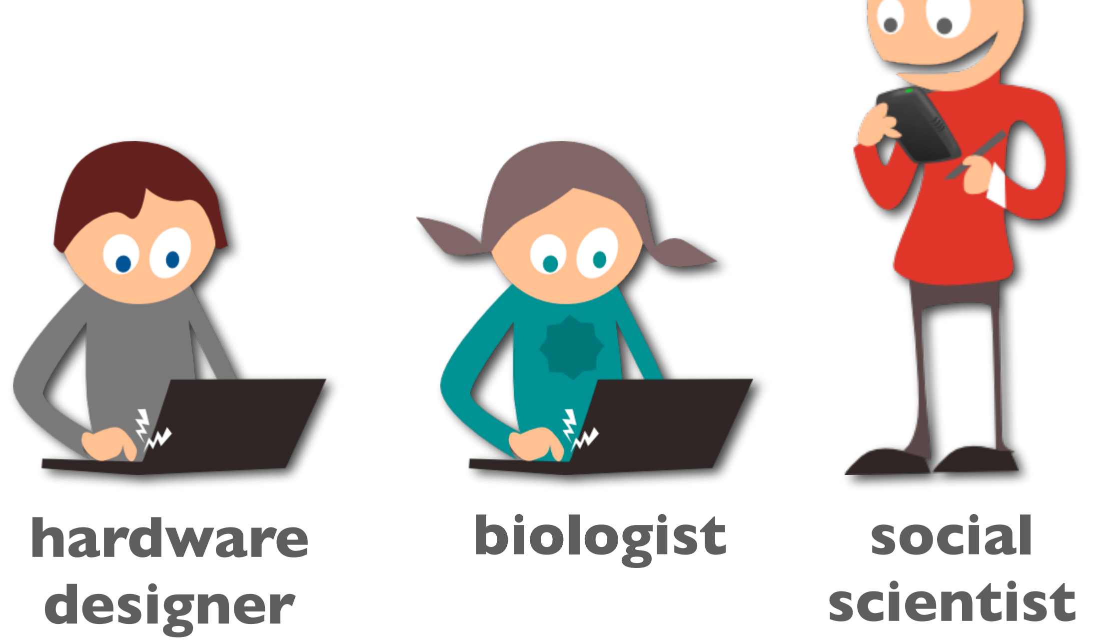
- spreadsheet data manipulation
- models of cell fates



A little programming for everyone

We all want to build programs …

- spreadsheet data manipulation
- models of cell fates
- cache coherence protocols
- memory models



A little programming for everyone

We all want to build programs …

- spreadsheet data manipulation [Flashfill, **POPL'11**]
- models of cell fates [SBL, **POPL'13**]
- cache coherence protocols [Transit, **PLDI'13**]
- memory models [MemSAT, **PLDI'10**]

time

code

effort

hardware designer

biologist

social scientist

A little programming for everyone

We all want to build programs …

- spreadsheet data manipulation
- models of cell fates
- cache coherence protocols
- memory models

solver-aided languages

less time

less code

less effort

hardware designer

biologist

social scientist

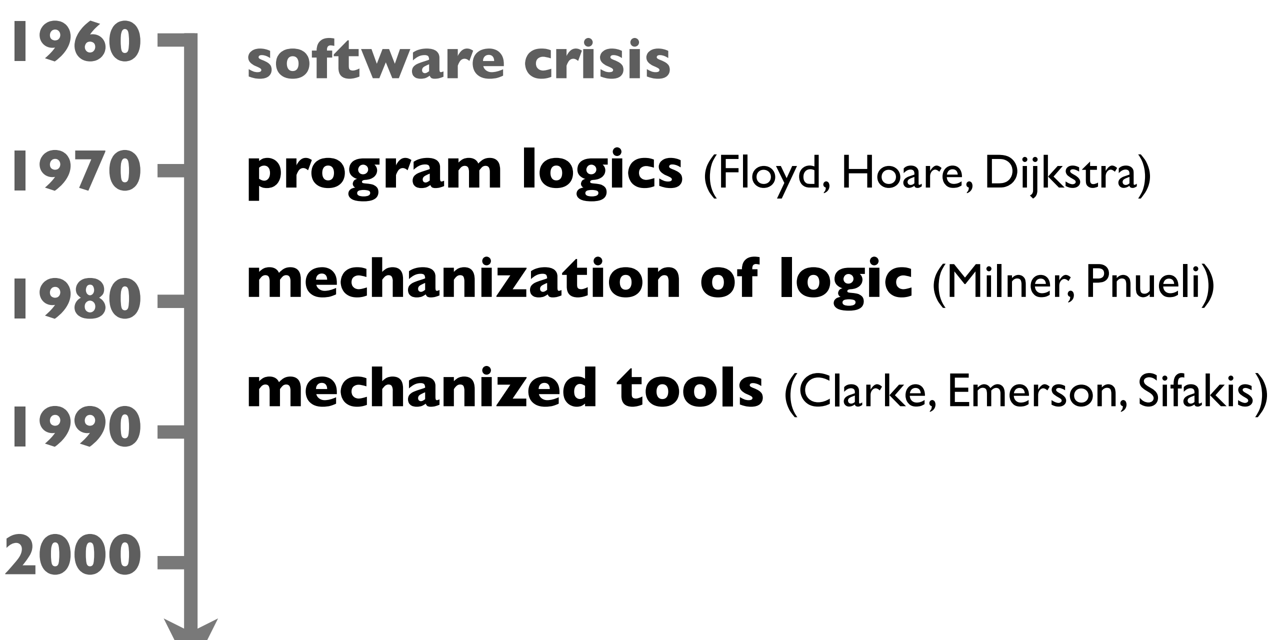
A little history

program logics (Floyd, Hoare, Dijkstra)

mechanization of logic (Milner, Pnueli)

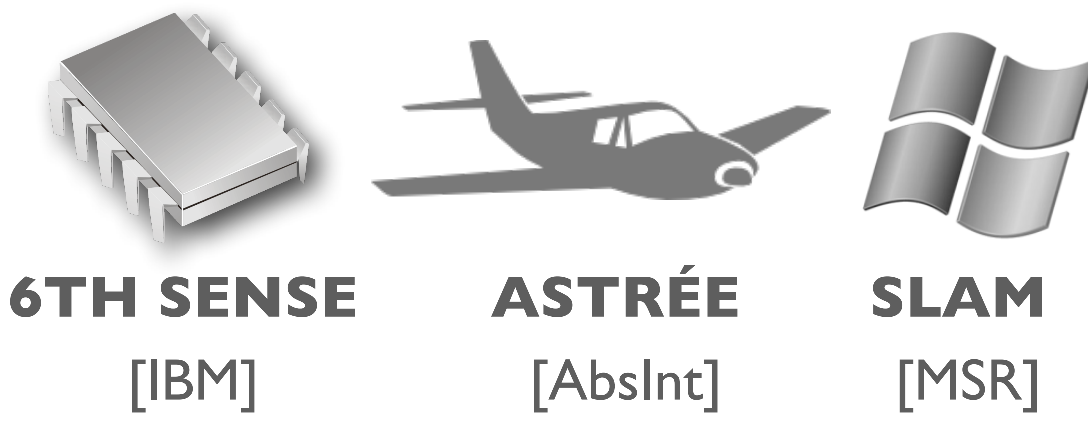
mechanized tools (Clarke, Emerson, Sifakis)

A little history

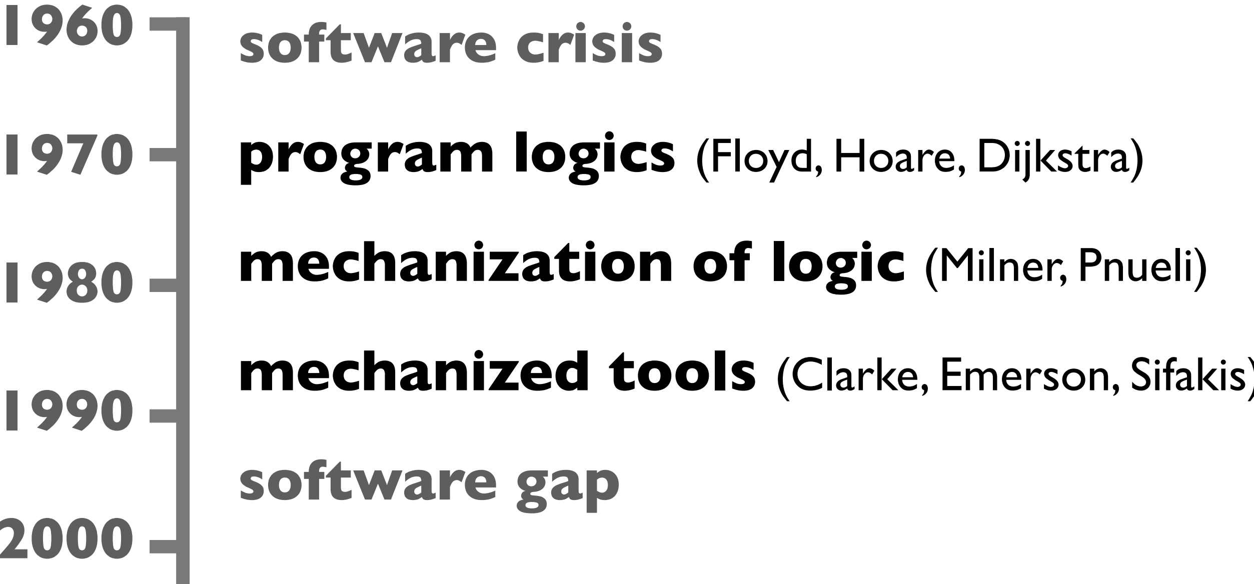


A little history

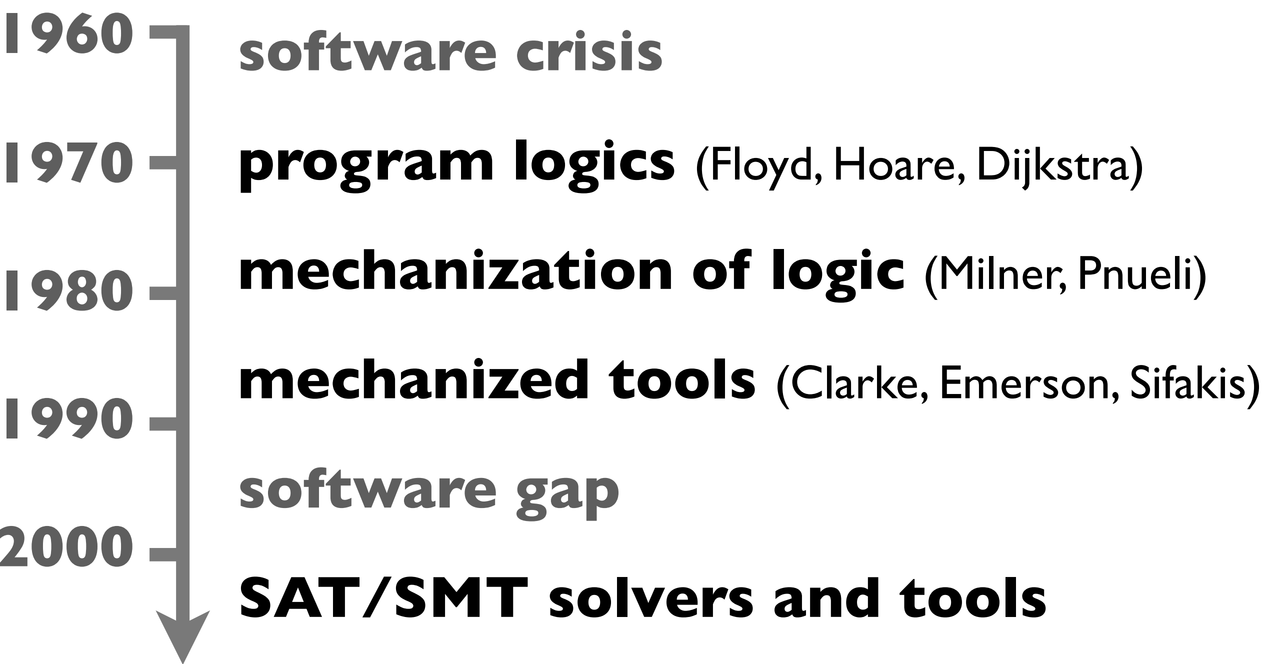
- 1960 — **software crisis**
- 1970 — **program logics** (Floyd, Hoare, Dijkstra)
- 1980 — **mechanization of logic** (Milner, Pnueli)
- 1990 — **mechanized tools** (Clarke, Emerson, Sifakis)
- 2000



A little history

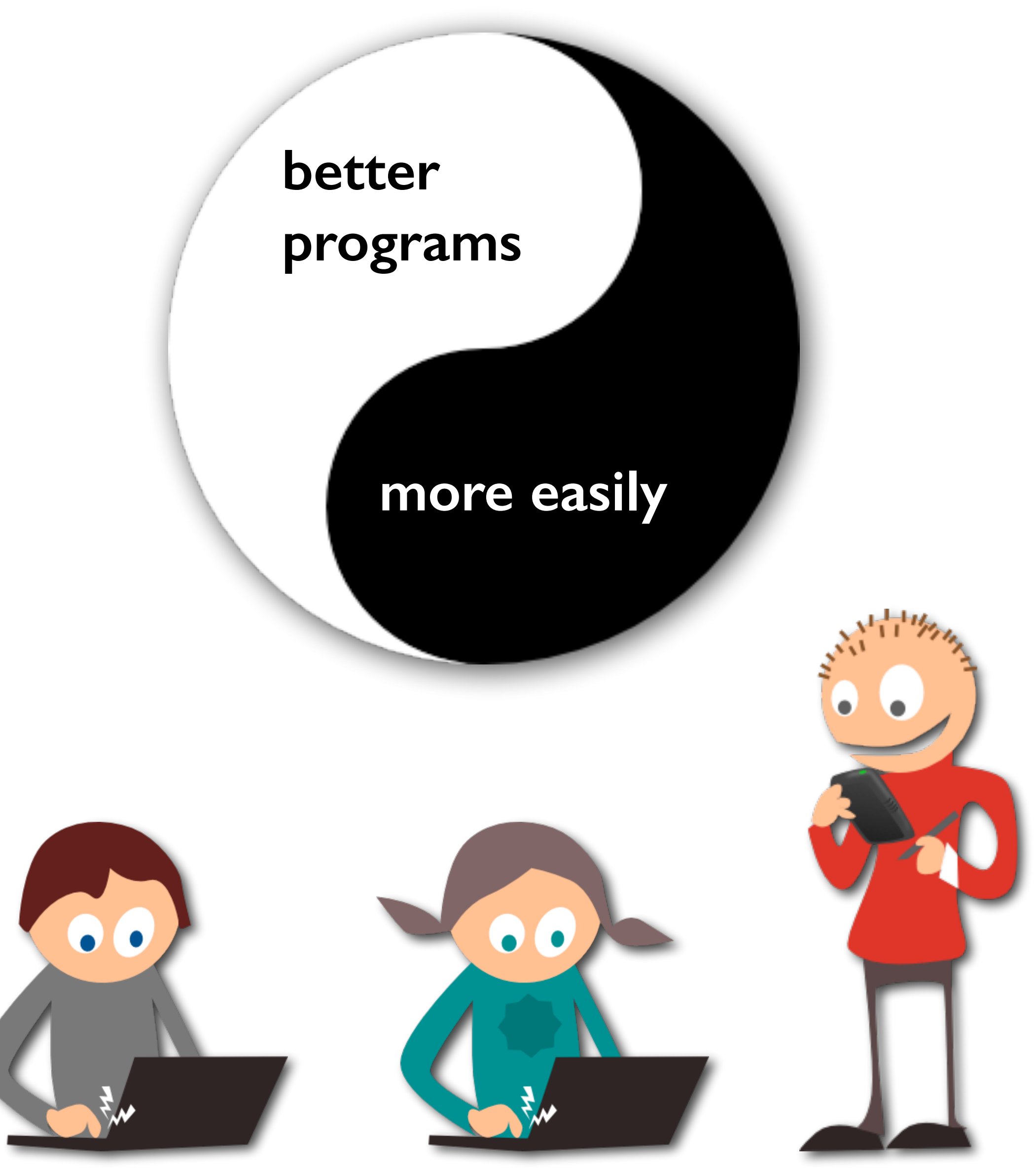


A little history



A little history

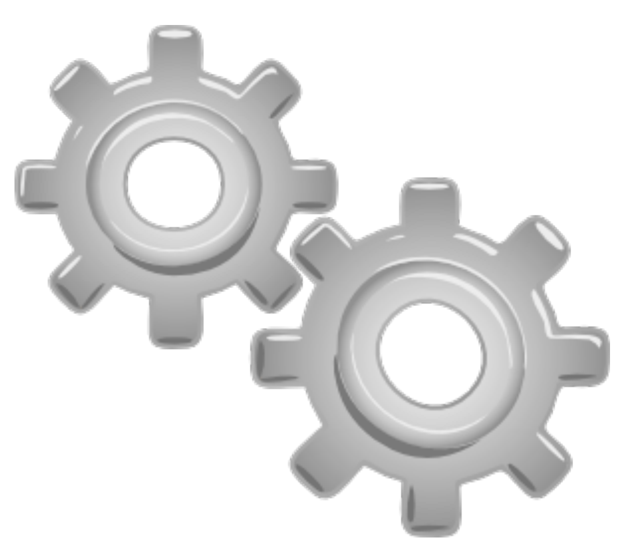
1960	**software crisis**
1970	**program logics** (Floyd, Hoare, Dijkstra)
1980	**mechanization of logic** (Milner, Pnueli)
	mechanized tools (Clarke, Emerson, Sifakis)
1990	**software gap**
2000	**SAT/SMT solvers and tools**
2010	**solver-aided languages**



outline

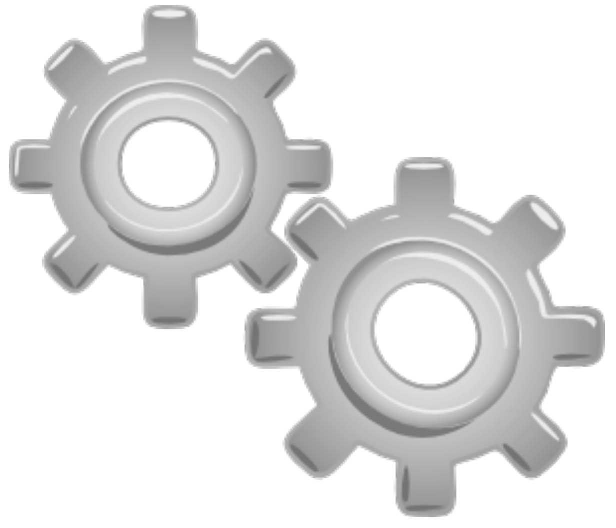
outline

solver-aided tools



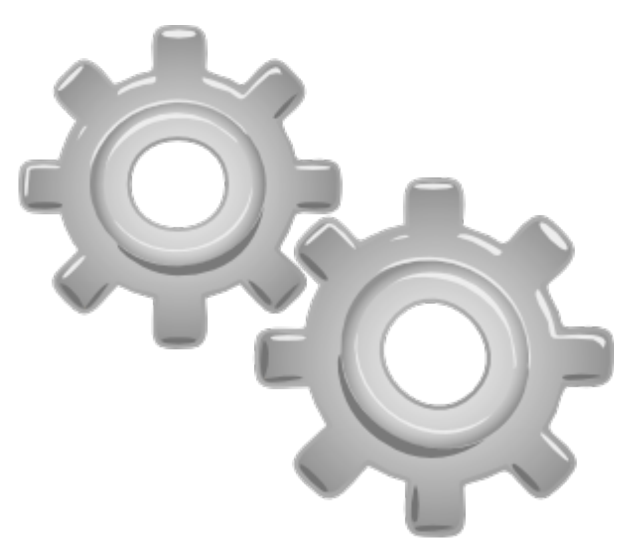
outline

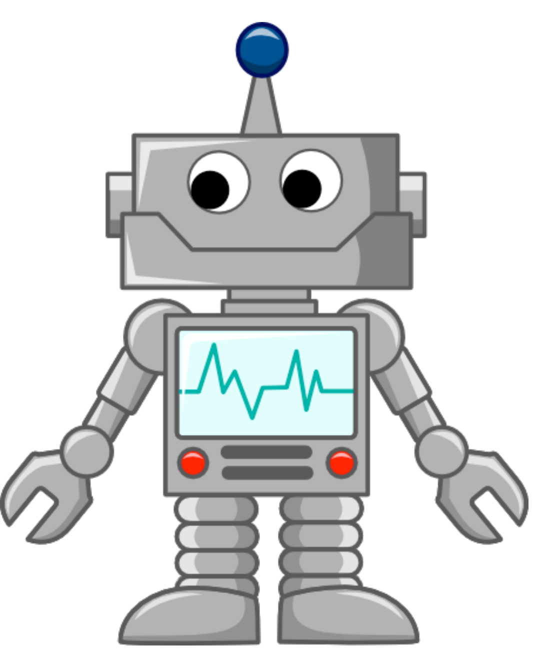
solver-aided tools, languages



outline

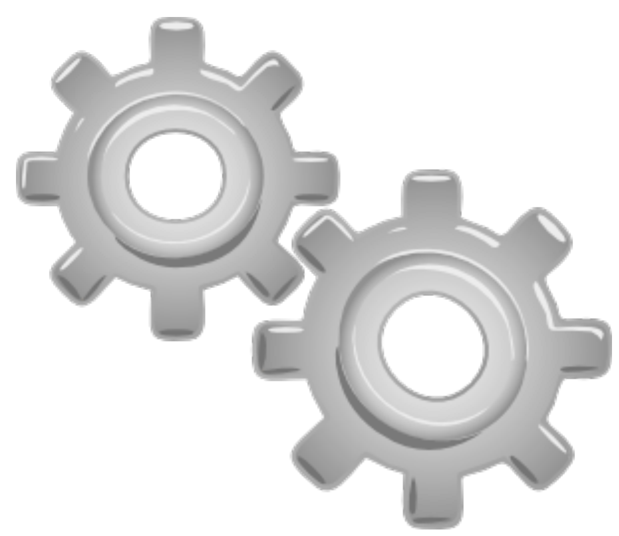
solver-aided tools, languages and demos



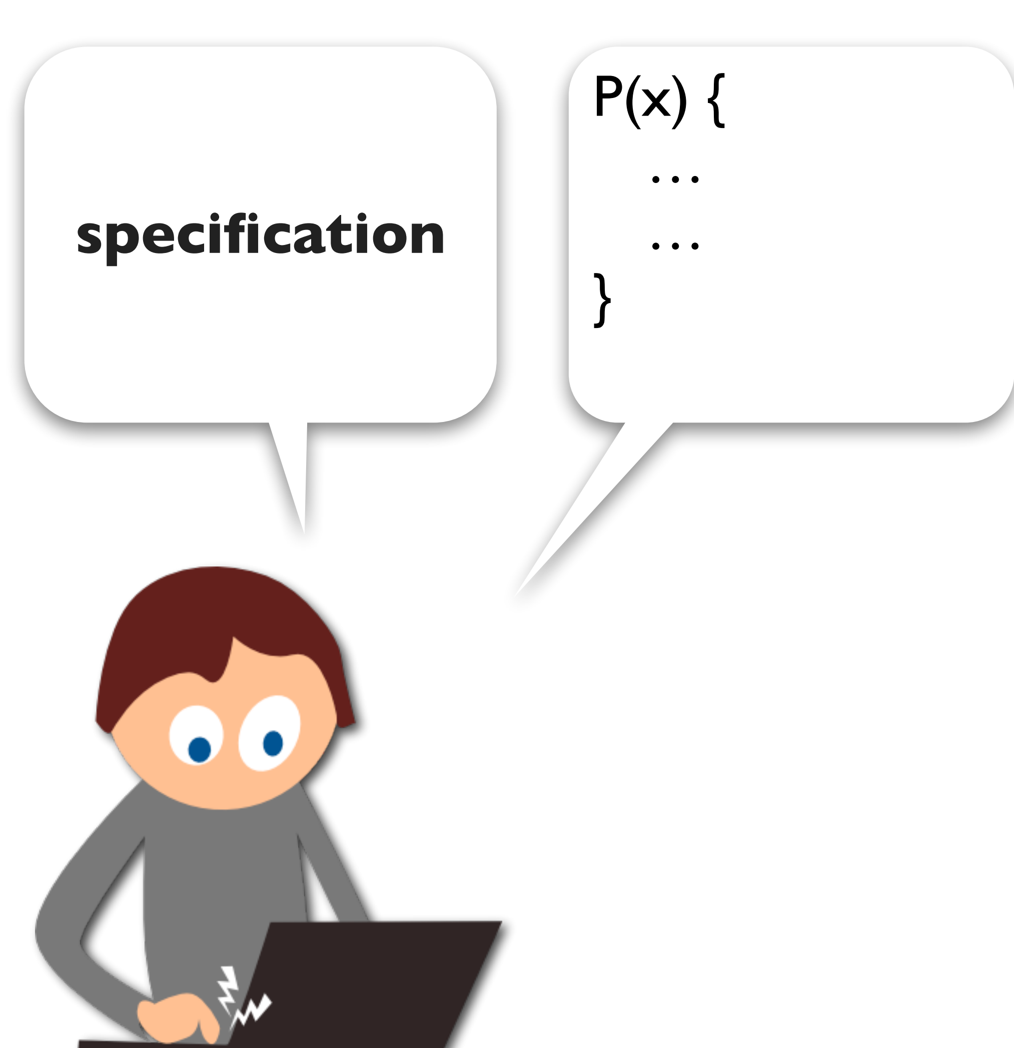


story

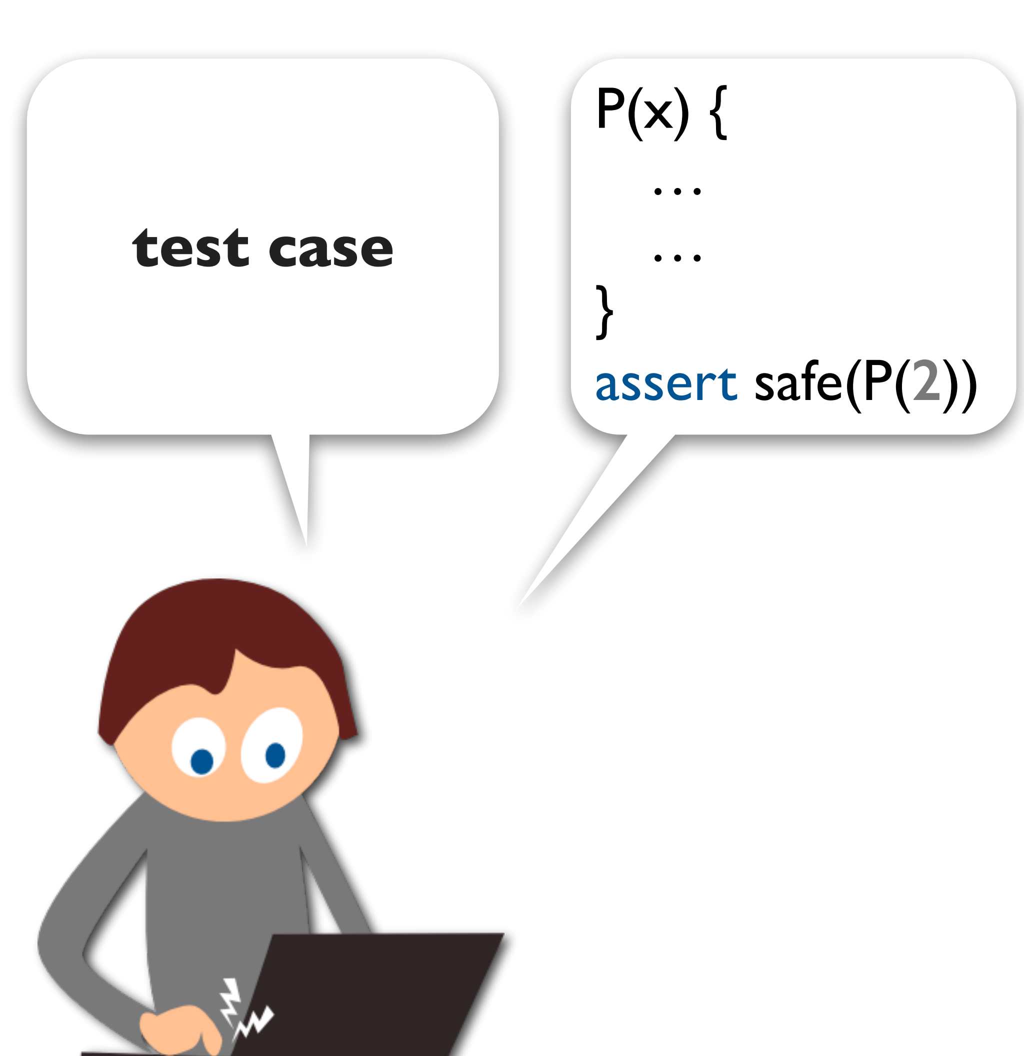
solver-aided tools



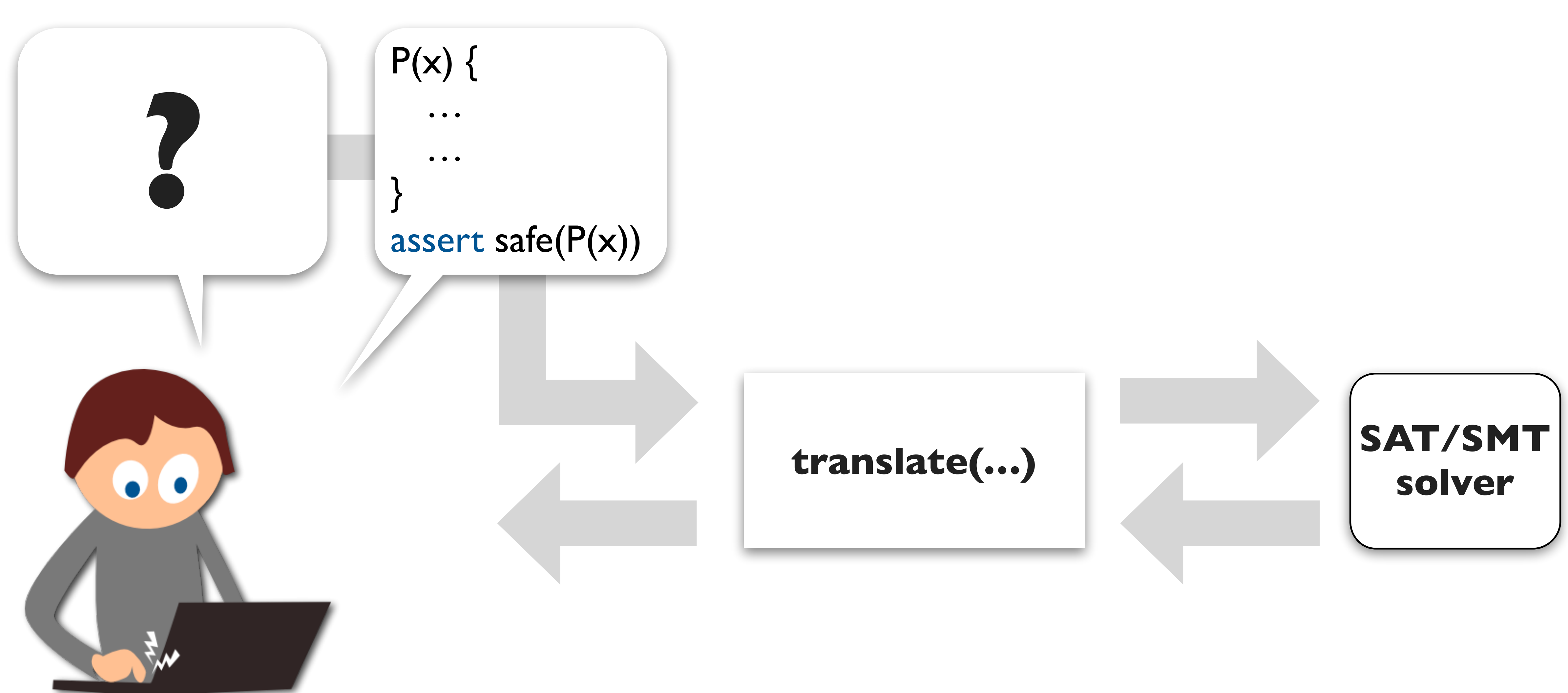
Programming ...



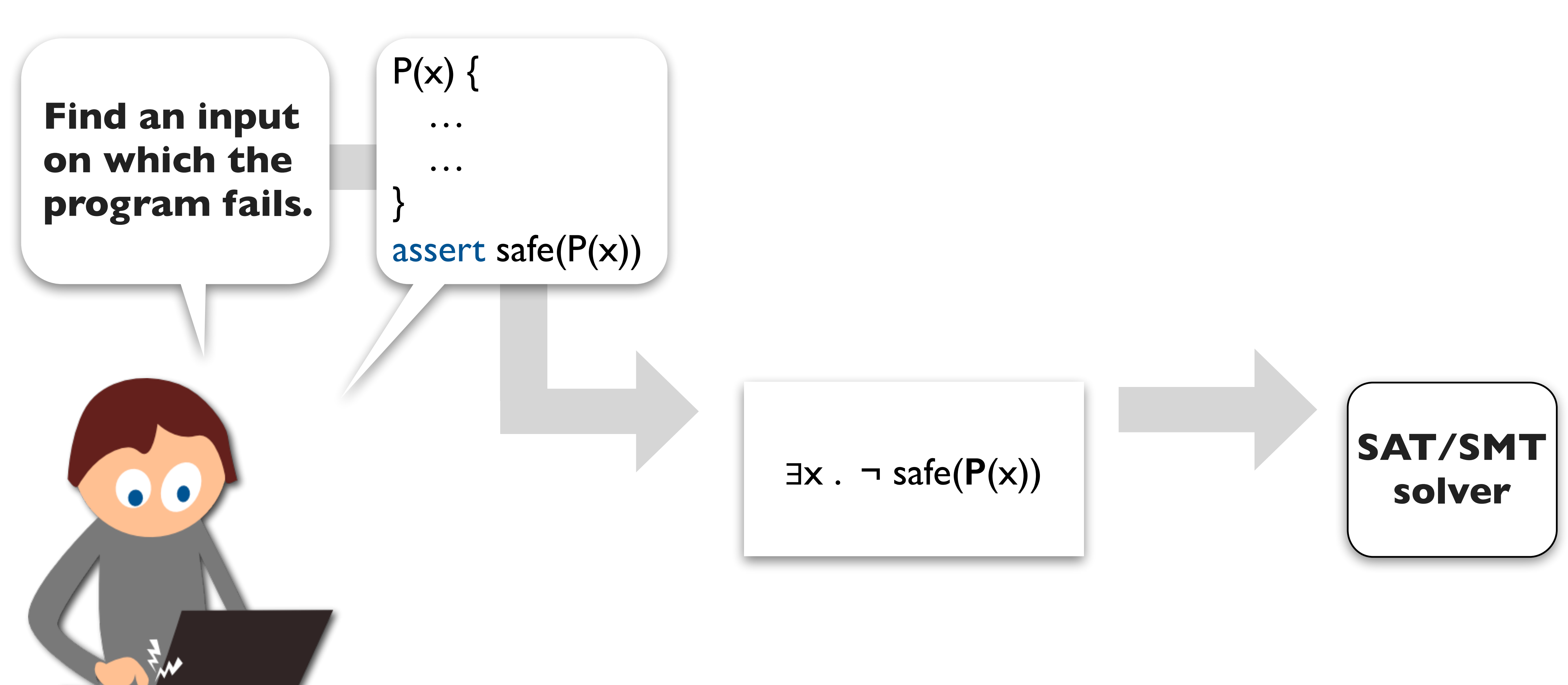
Programming ...



Programming with a solver-aided tool

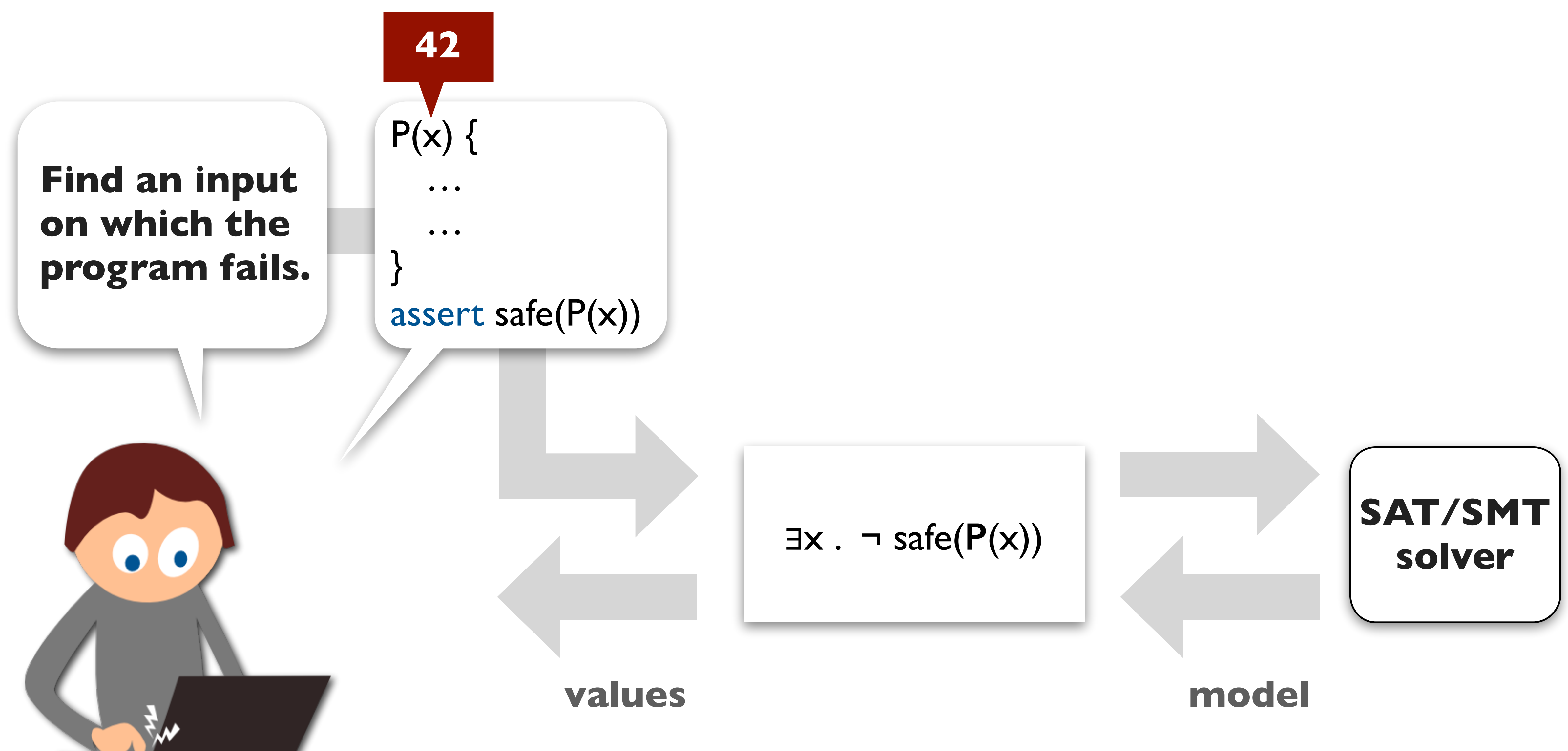


Solver-aided tools: verification



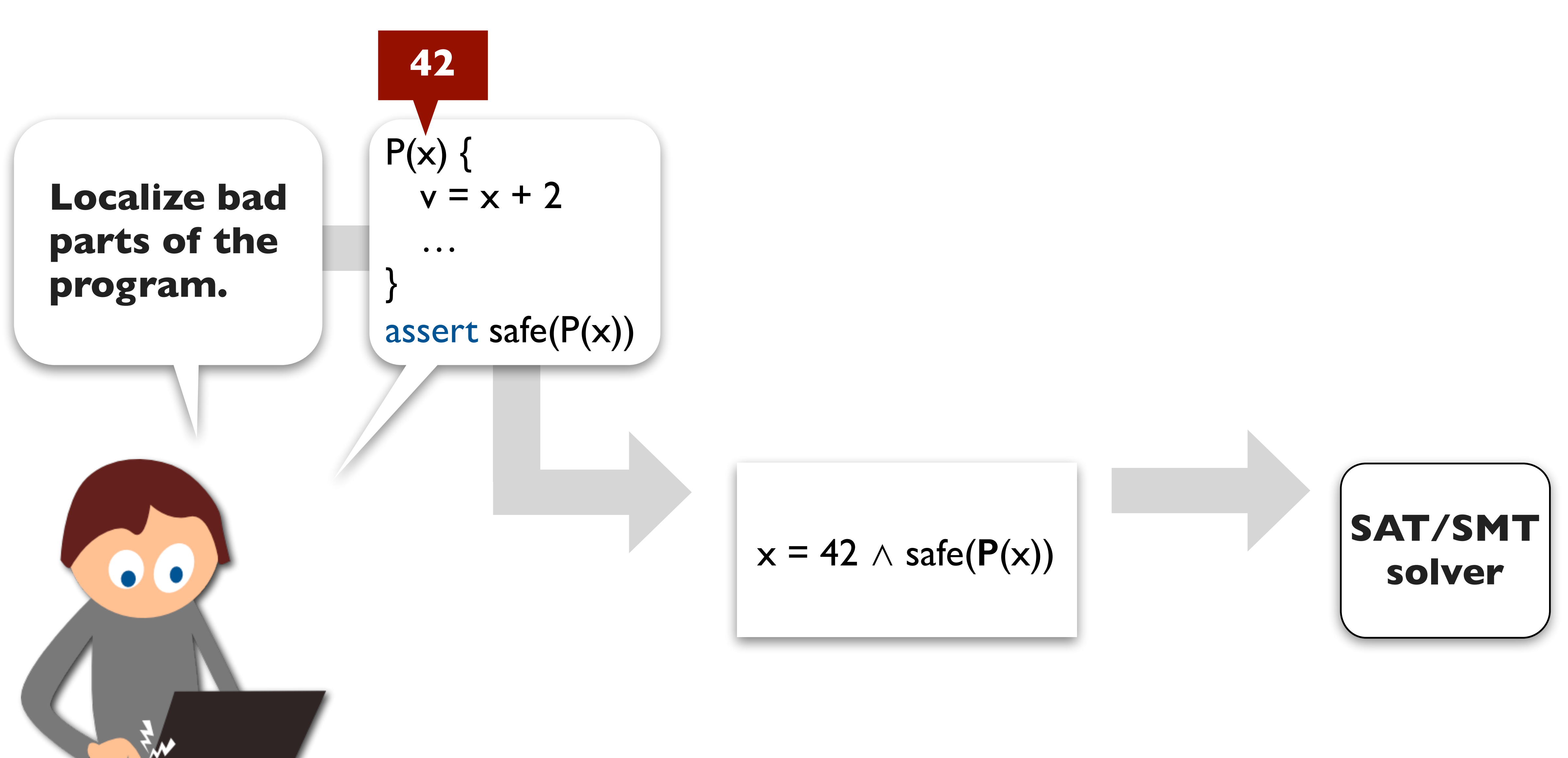
CBMC [CMU], Dafny [MSR], Miniatur [IBM], Klee [Stanford], etc.

Solver-aided tools: verification



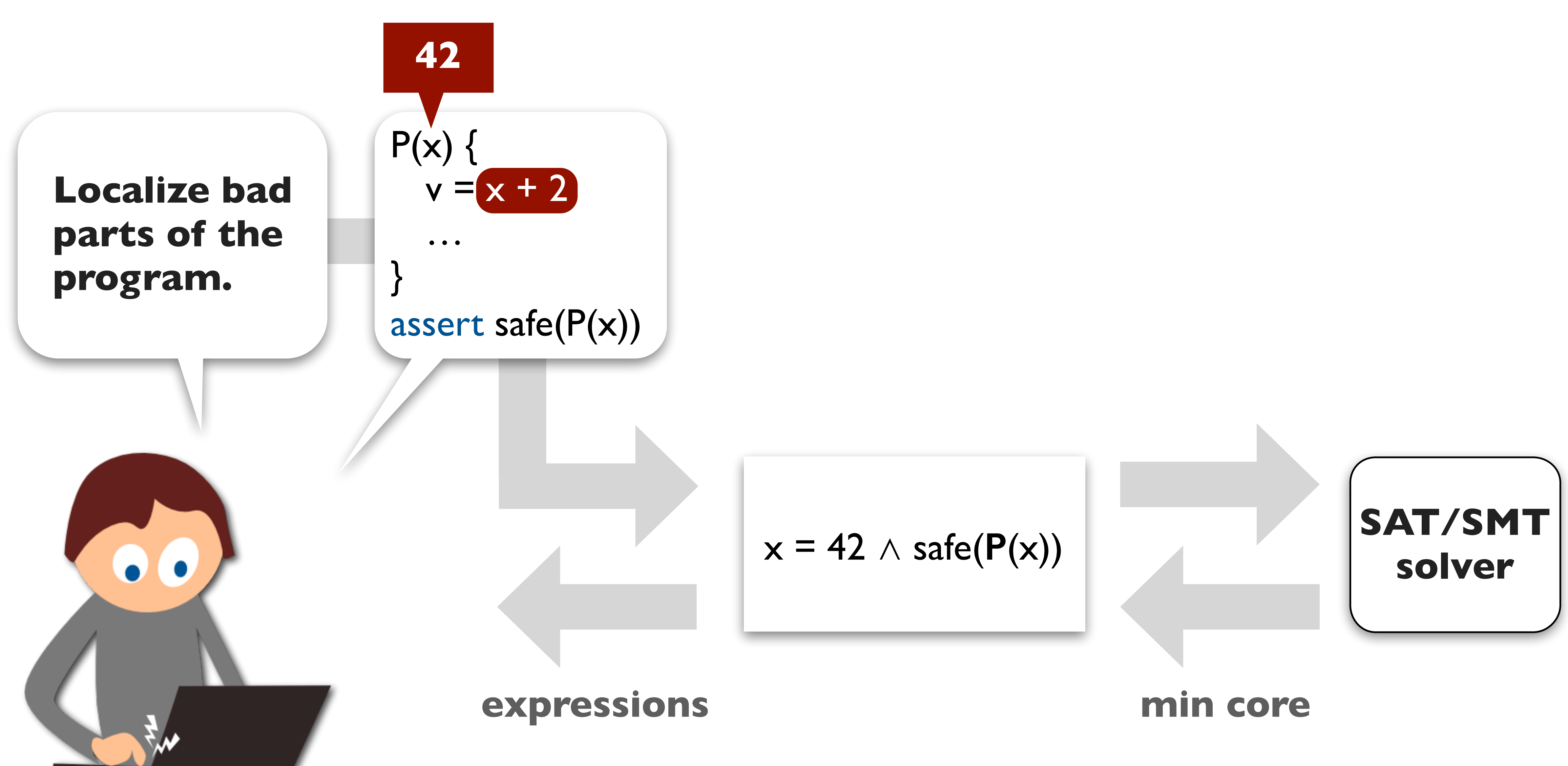
CBMC [CMU], Dafny [MSR],
Miniatur [IBM], Klee [Stanford], etc.

Solver-aided tools: debugging



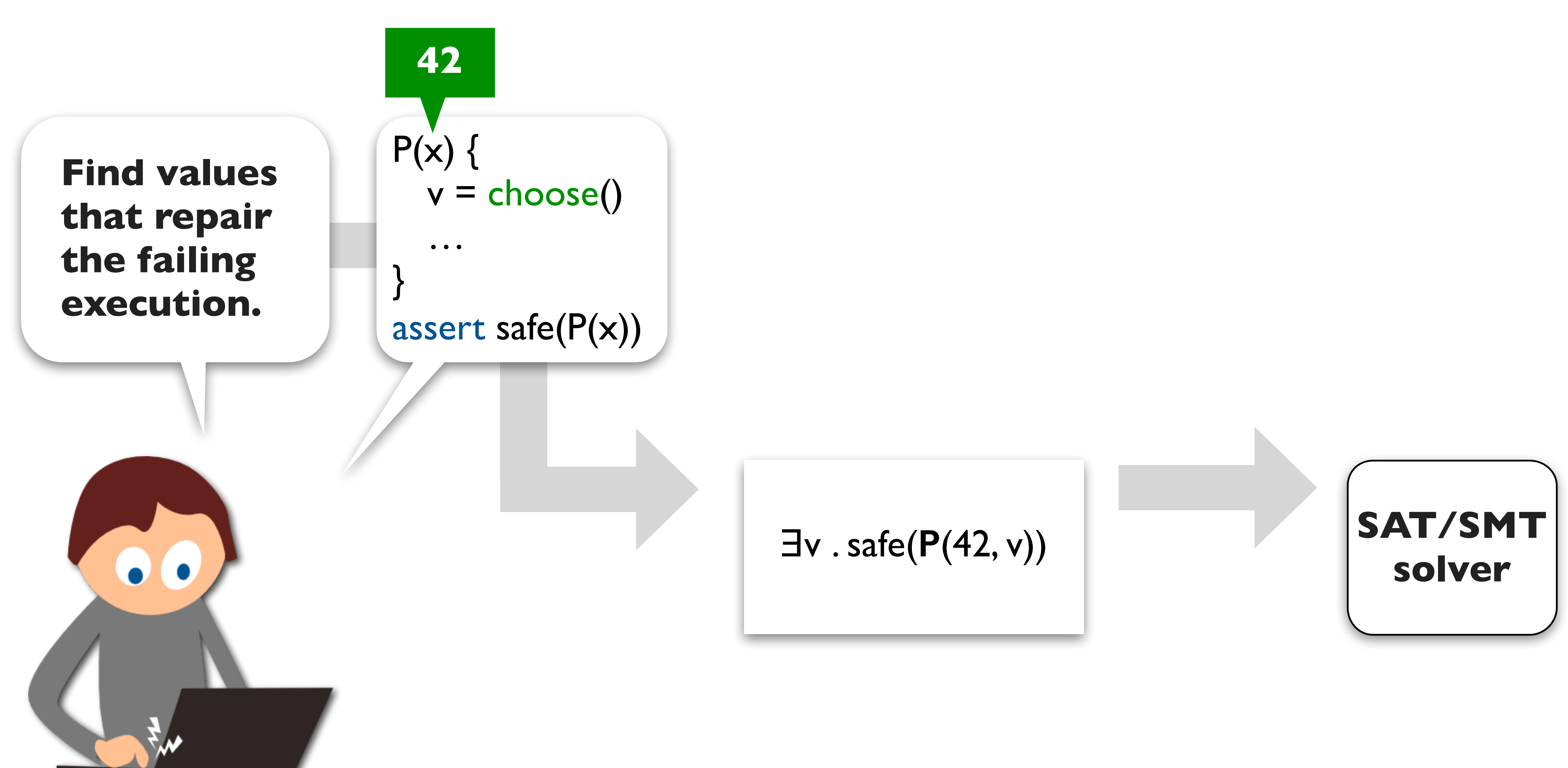
BugAssist [UCLA / MPI-SWS]

Solver-aided tools: debugging



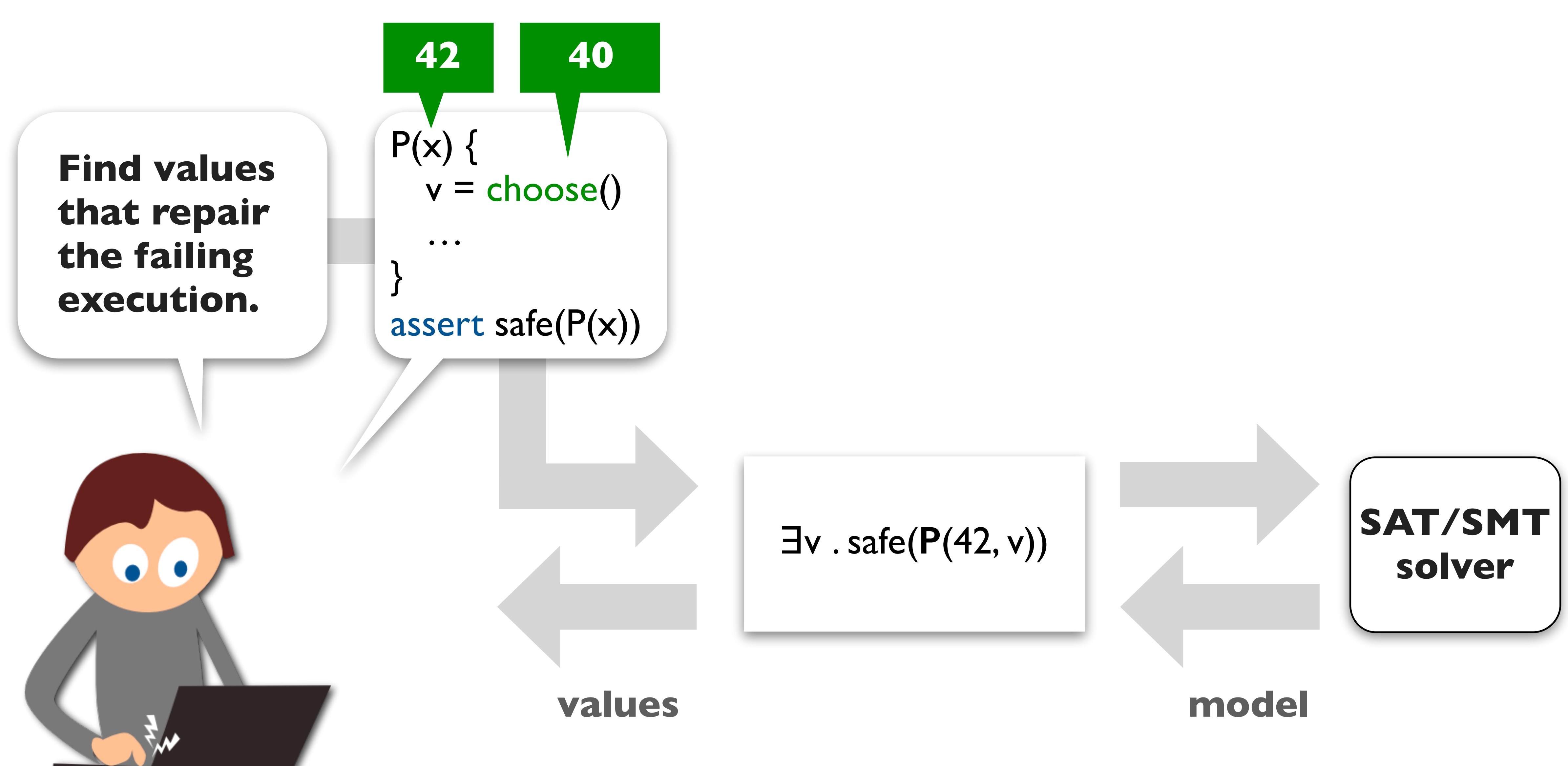
BugAssist [UCLA / MPI-SWS]

Solver-aided tools: angelic execution



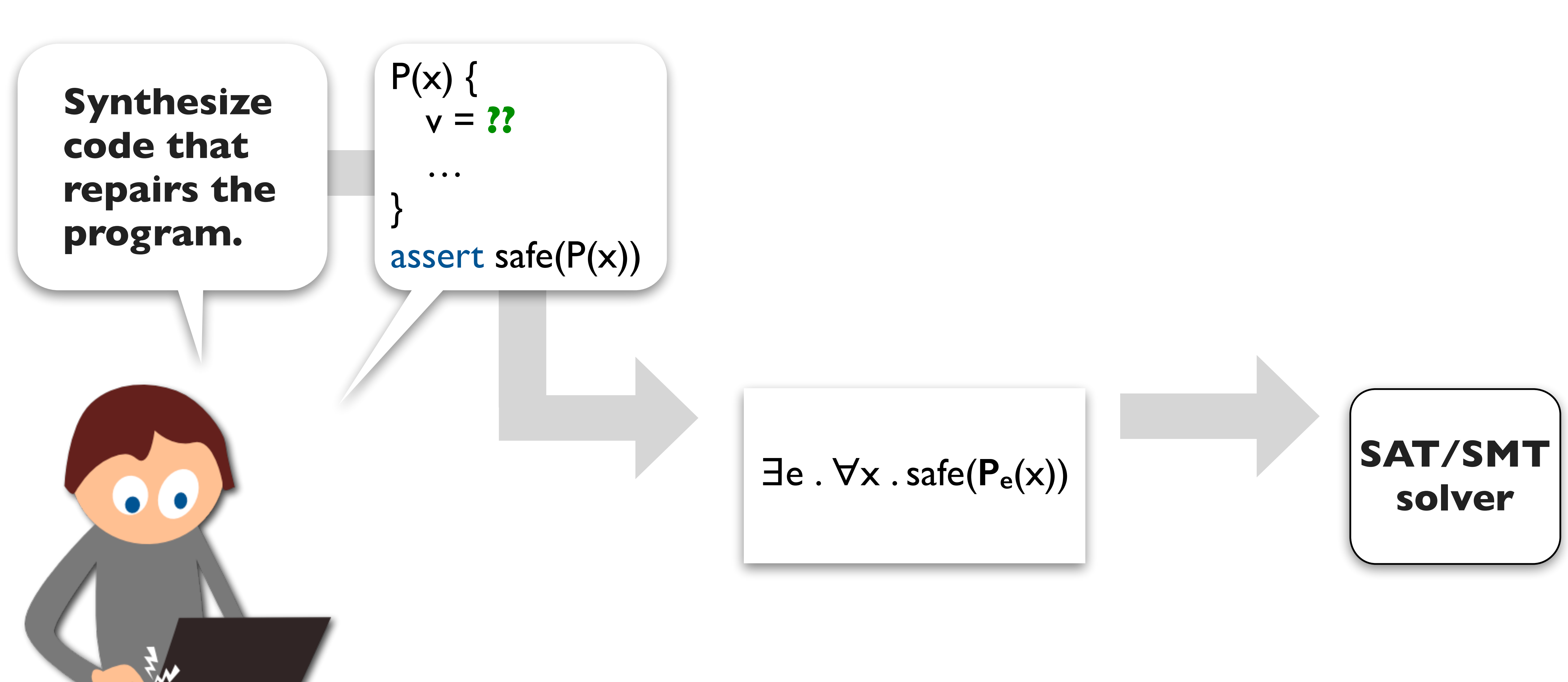
Kaplan [EPFL], PBnJ [UCLA], Skalch [Berkeley], Squander [MIT], etc.

Solver-aided tools: angelic execution



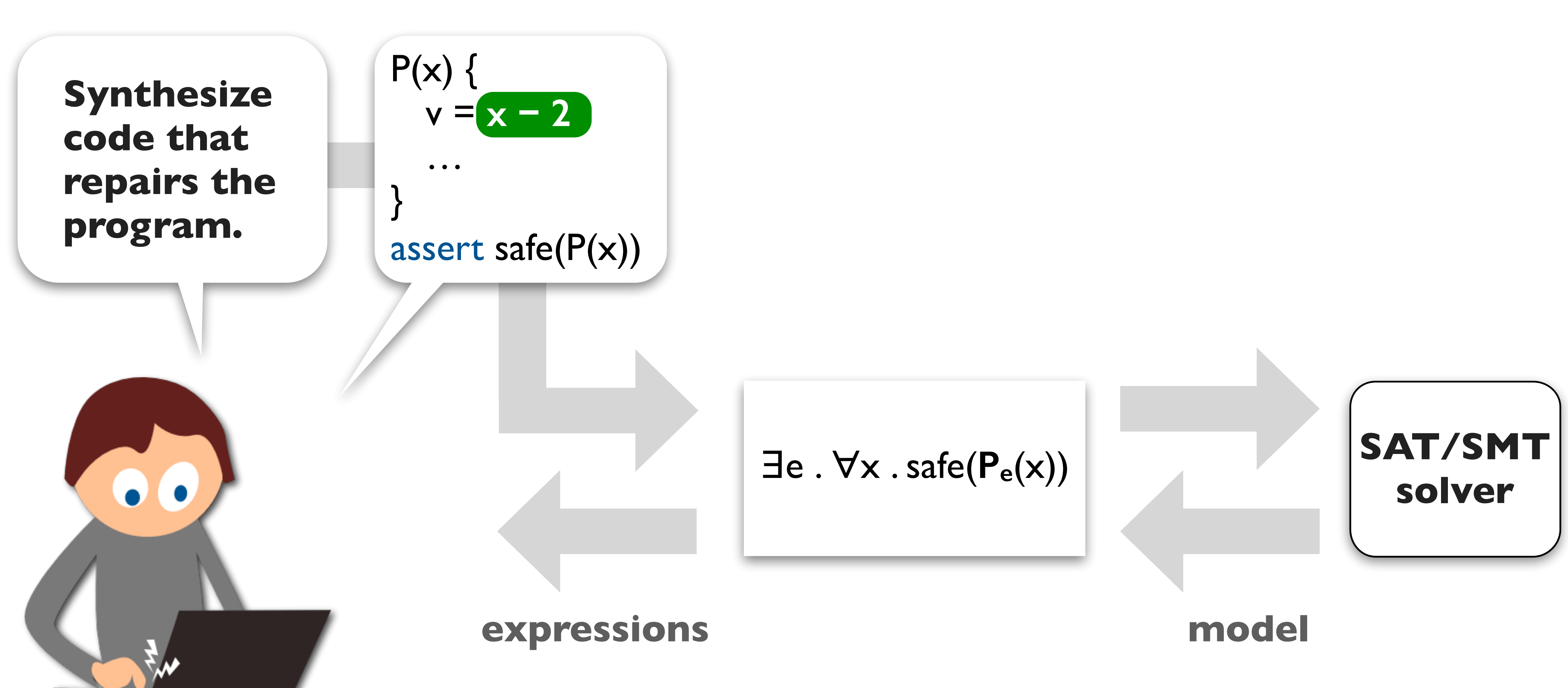
Kaplan [EPFL], PBnJ [UCLA], Skalch [Berkeley], Squander [MIT], etc.

Solver-aided tools: synthesis



Comfusy [EPFL], Sketch [Berkeley / MIT]

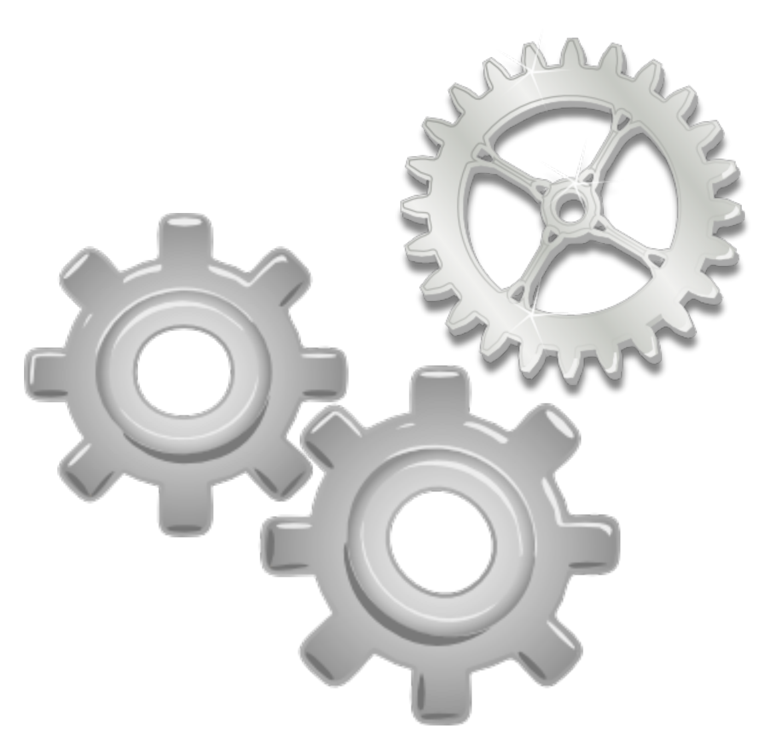
Solver-aided tools: synthesis



Comfusy [EPFL], Sketch [Berkeley / MIT]

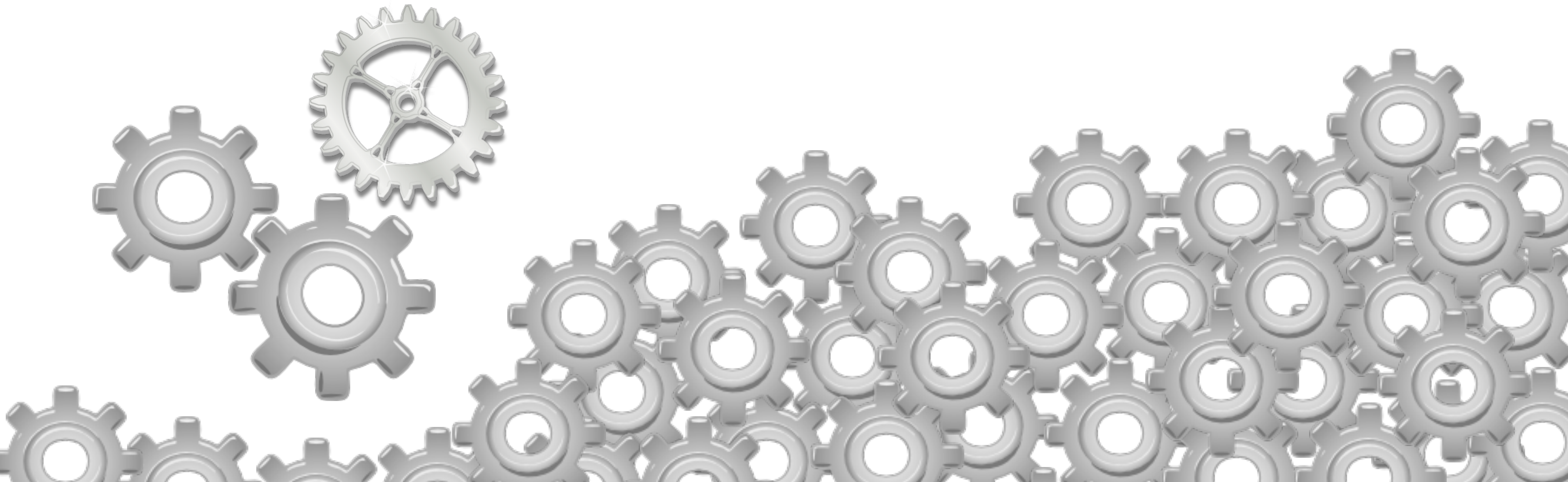
wanted

more solver-aided tools …

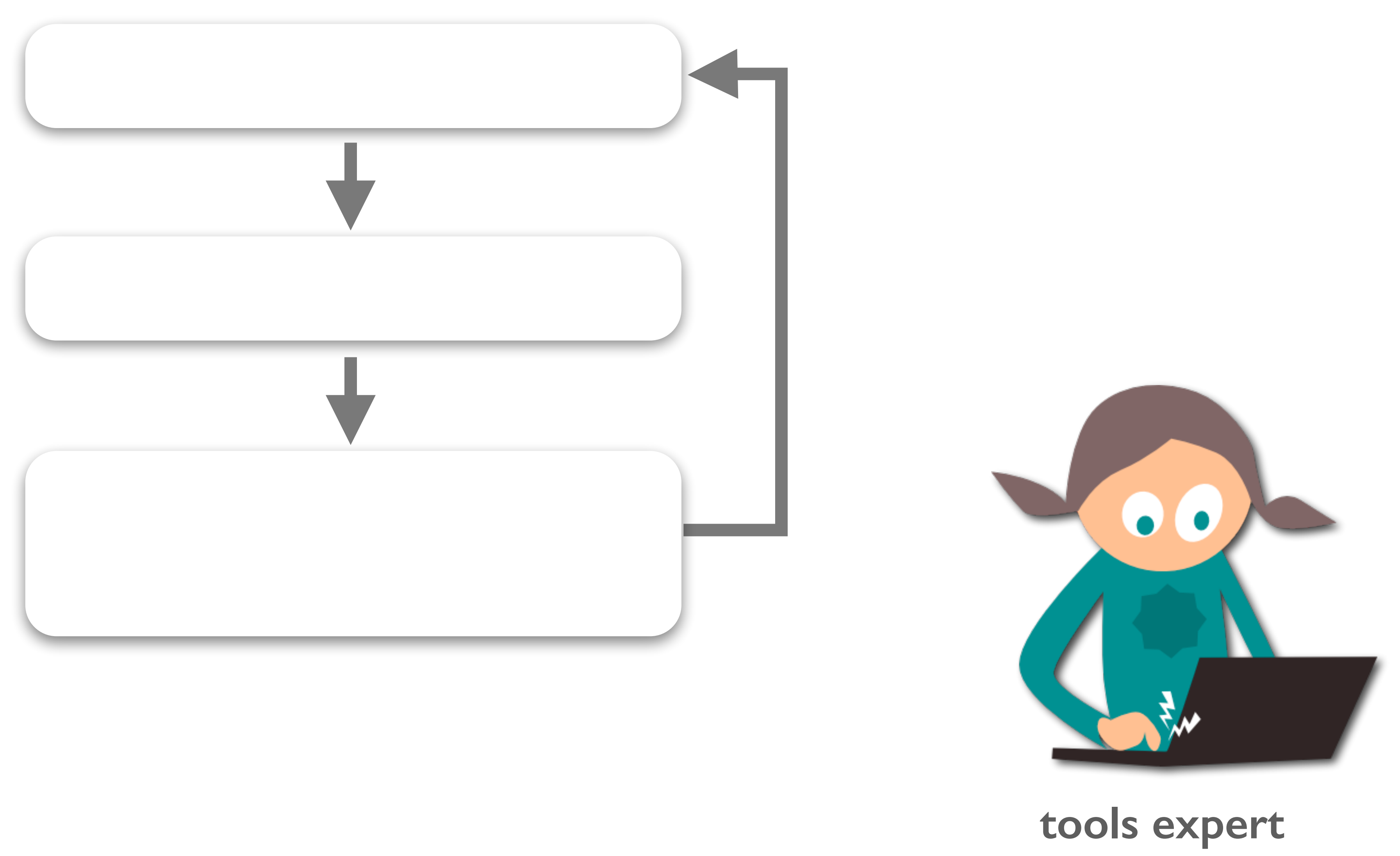


wanted

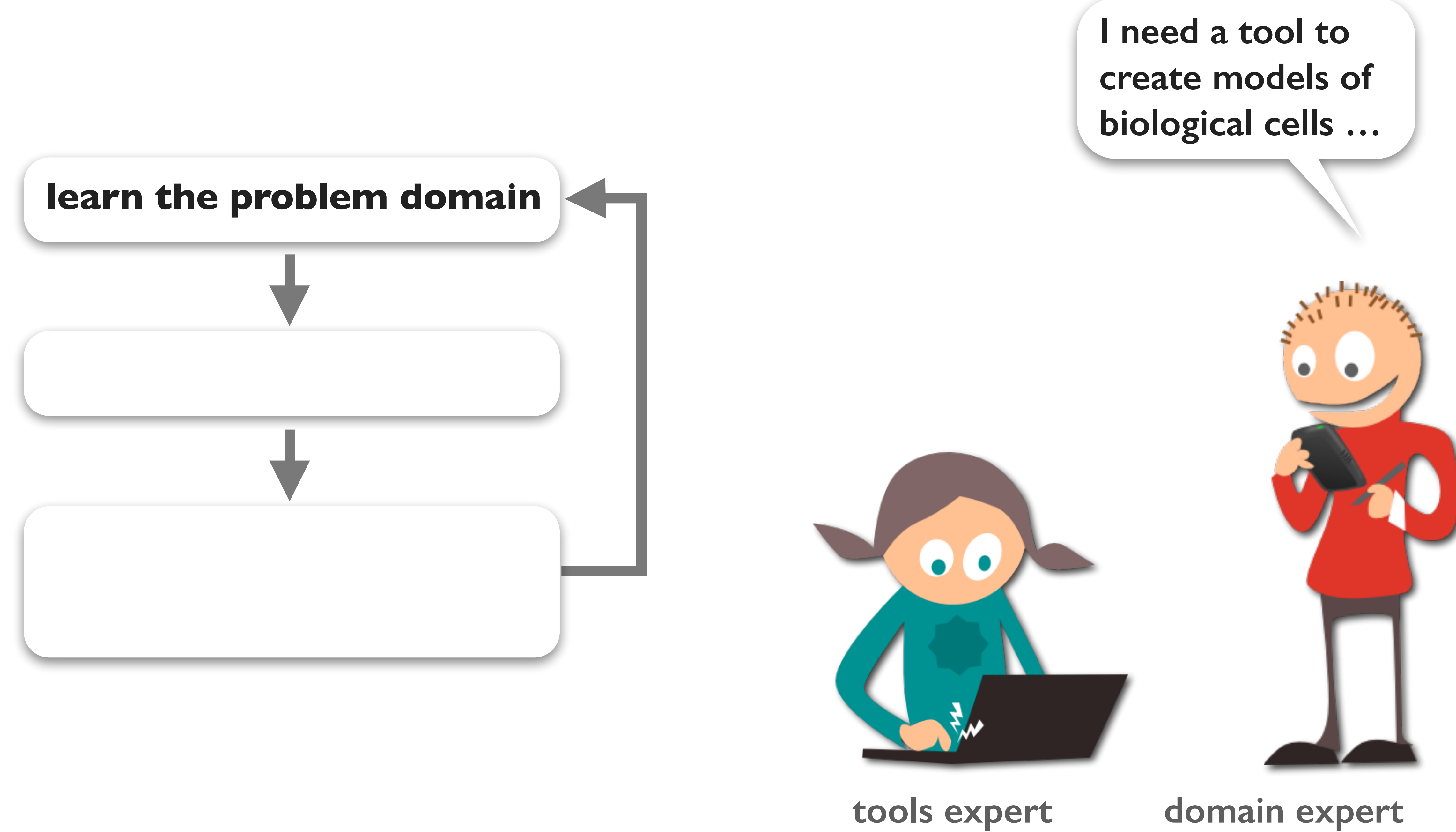
more solver-aided tools …



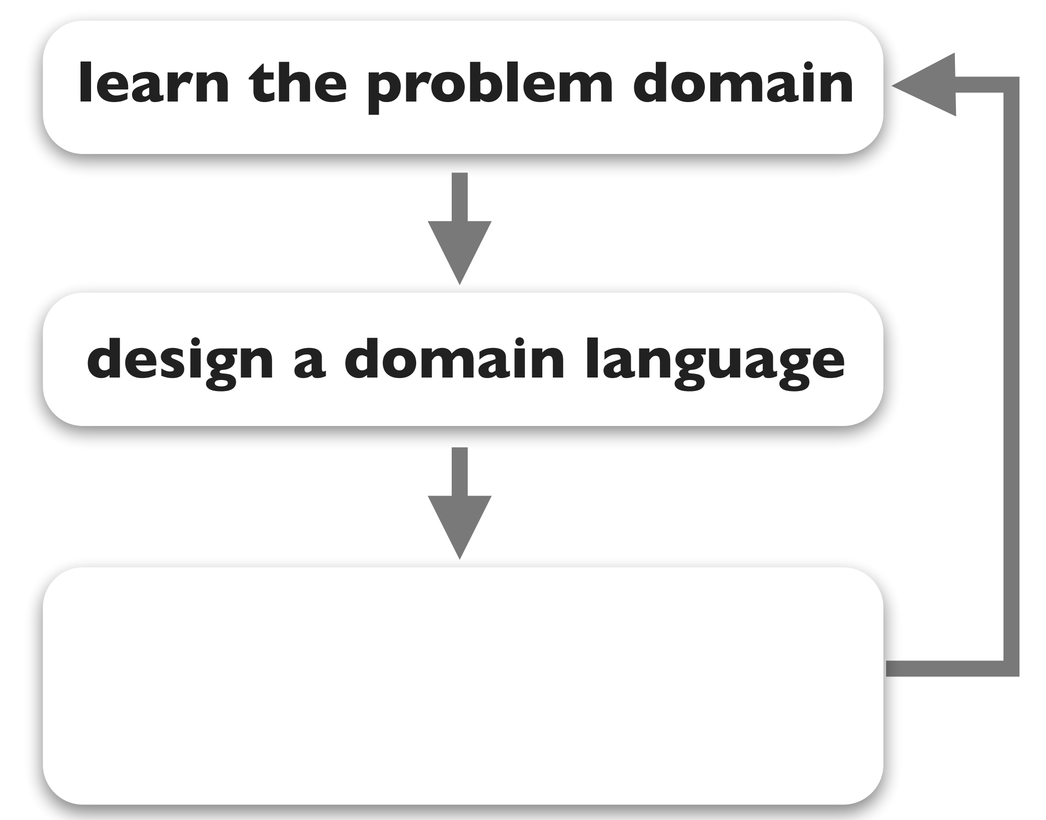
Building solver-aided tools: state-of-the-art

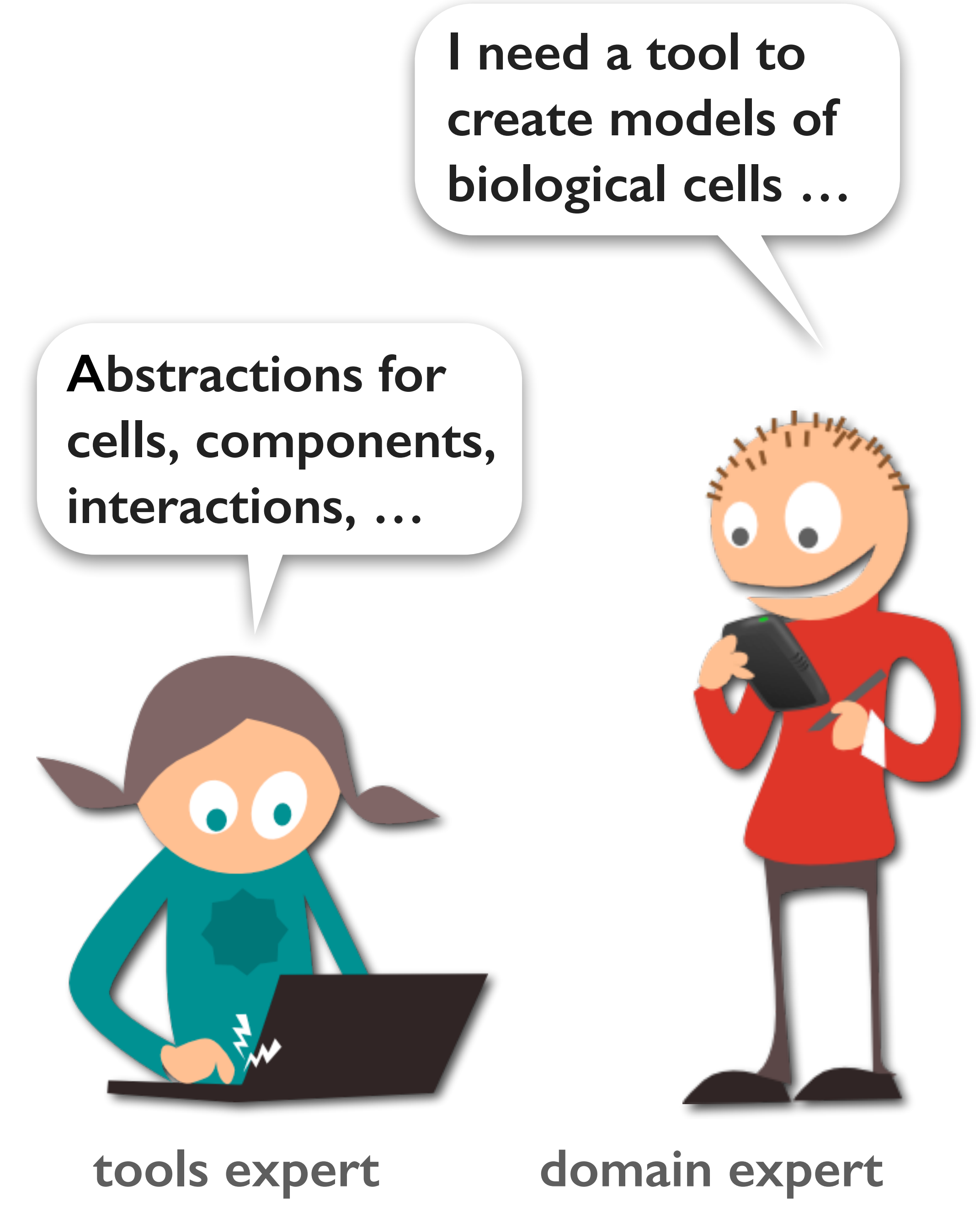


Building solver-aided tools: state-of-the-art

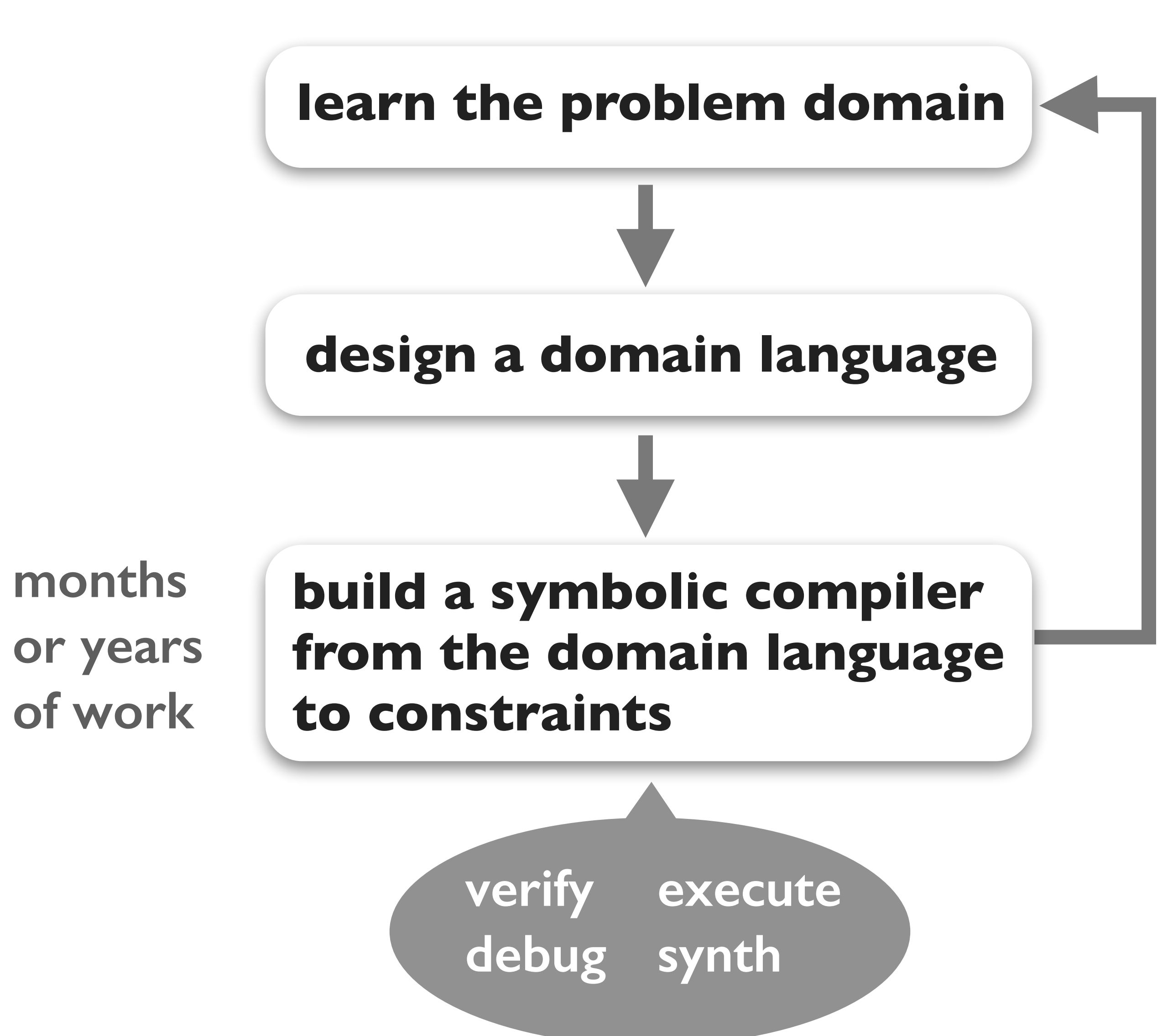


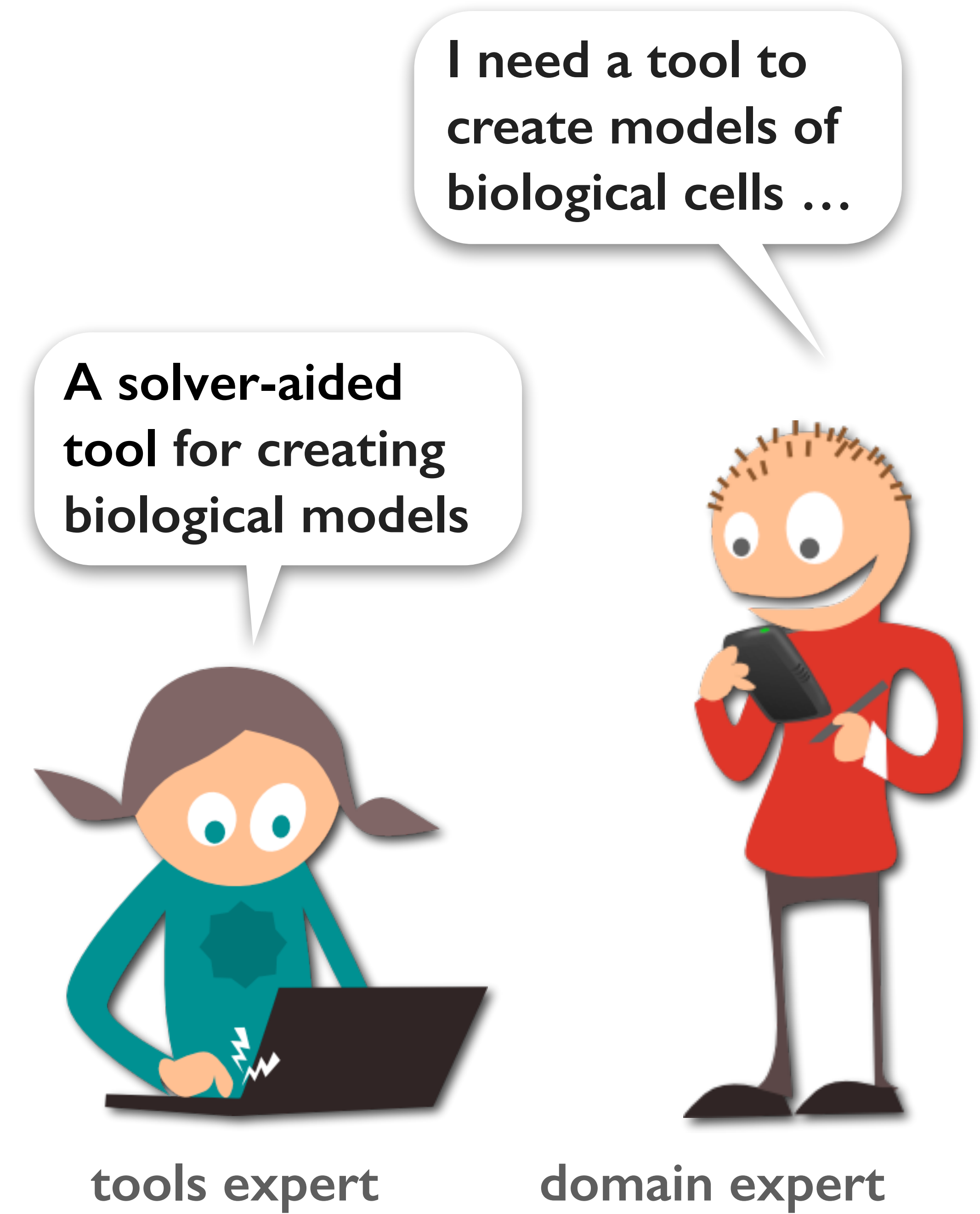
Building solver-aided tools: state-of-the-art



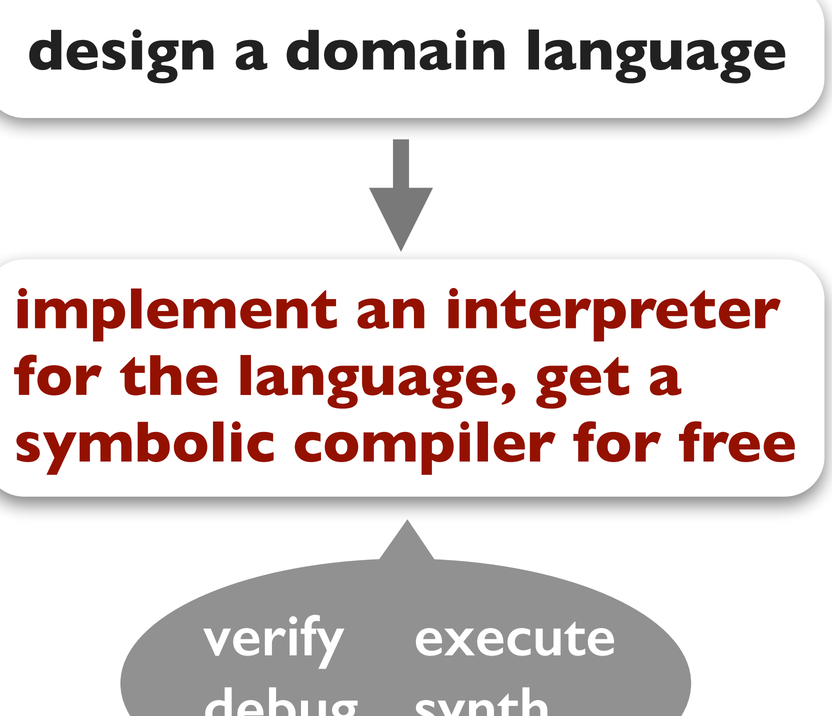


Building solver-aided tools: state-of-the-art





Can we do better?



Can we do better?

a solver-aided domain-specific language (SDSL)

design a domain language

implement an interpreter for the language, get a symbolic compiler for free

weeks

verify execute
debug synth

domain expert

design

solver-aided languages

Layers of languages

domain-specific language (DSL)

library

interpreter

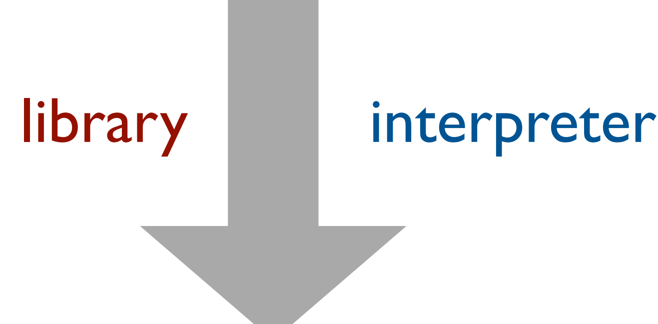
host language

A formal language that is specialized to a particular application domain and often limited in capability.

A high-level language for implementing DSLs, usually with meta-programming features.

Layers of languages

domain-specific language
(DSL)



host language

artificial intelligence
Church, BLOG

databases
SQL, Datalog

hardware design
Bluespec, Chisel, Verilog, VHDL

math and statistics
Eigen, Matlab, R

layout and visualization
LaTex, dot, dygraphs, D3

Scala, Racket, JavaScript

Layers of languages

domain-specific language
(DSL)

host language

Eigen / Matlab

```
C = A * B
```

[associativity]

C / Java

```
for (i = 0; i < n; i++)
 for (j = 0; j < m; j++)
  for (k = 0; k < p; k++)
   C[i][k] += A[i][j] * B[j][k]
```

Layers of solver-aided languages

solver-aided domain-specific language (SDSL)

library

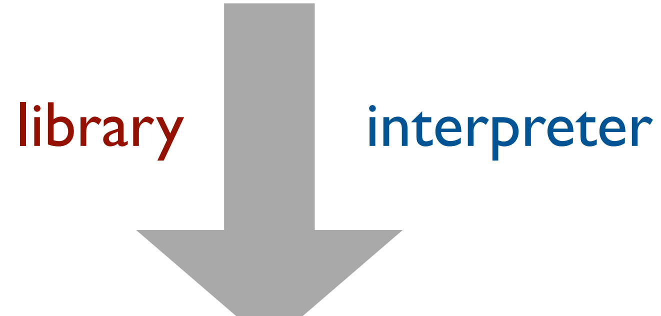
interpreter

solver-aided host language

symbolic virtual machine

Layers of solver-aided languages

solver-aided **domain-specific language (SDSL)**



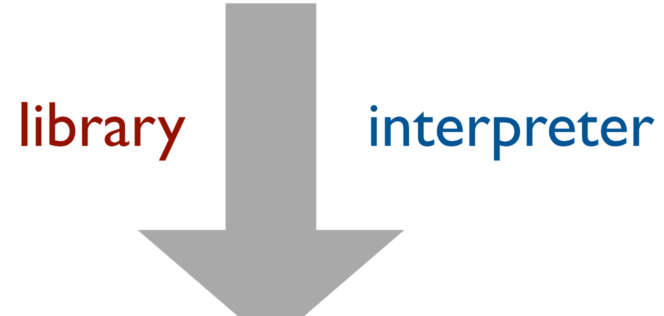
solver-aided **host language**

symbolic virtual machine

[Torlak & Bodik, **Onward'13, PLDI'14**]

Layers of solver-aided languages

solver-aided **domain-specific language (SDSL)**



solver-aided **host language**

symbolic virtual machine

spatial programming
Chlorophyll

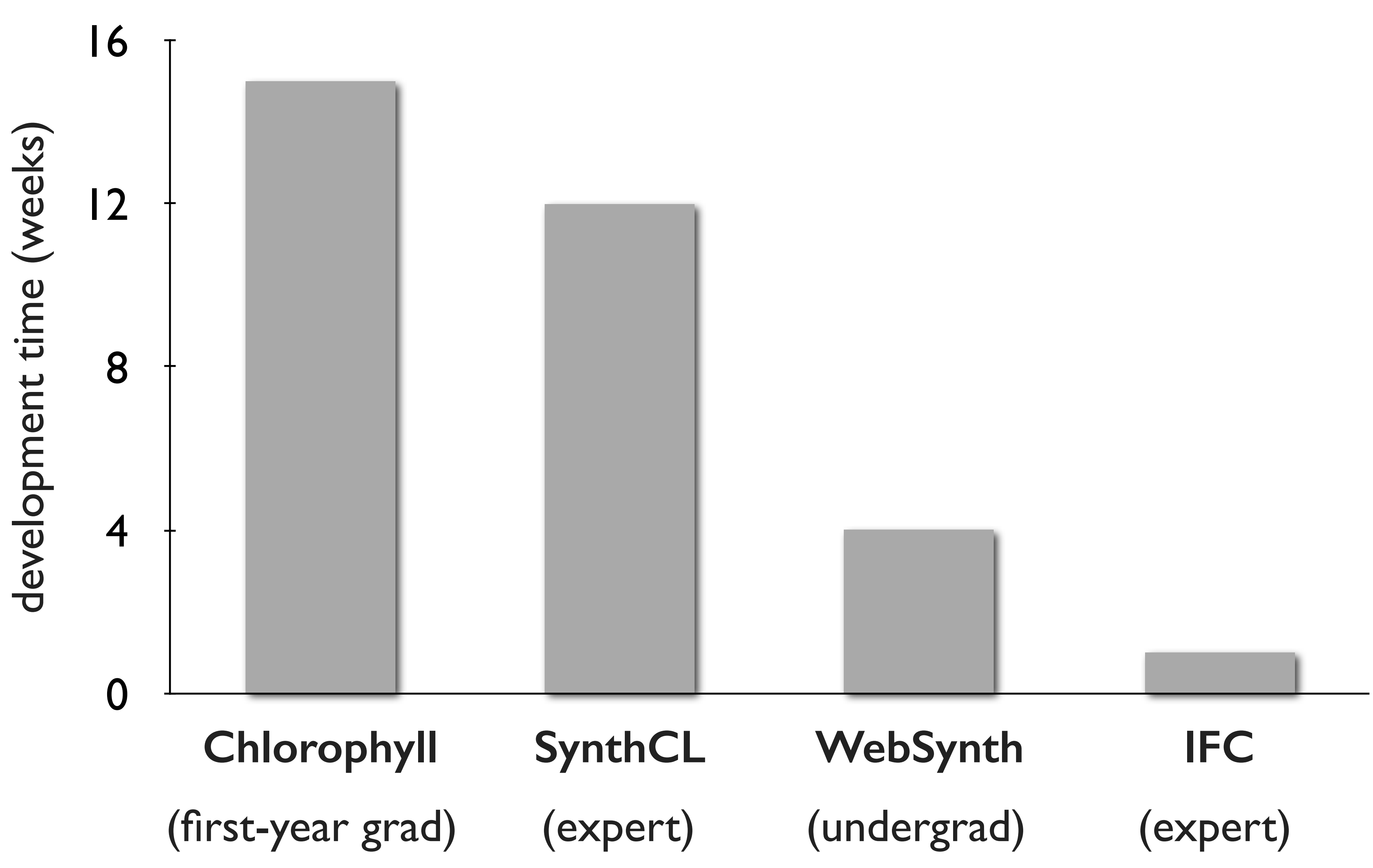
data-parallel programming
SynthCL

web scraping
WebSynth

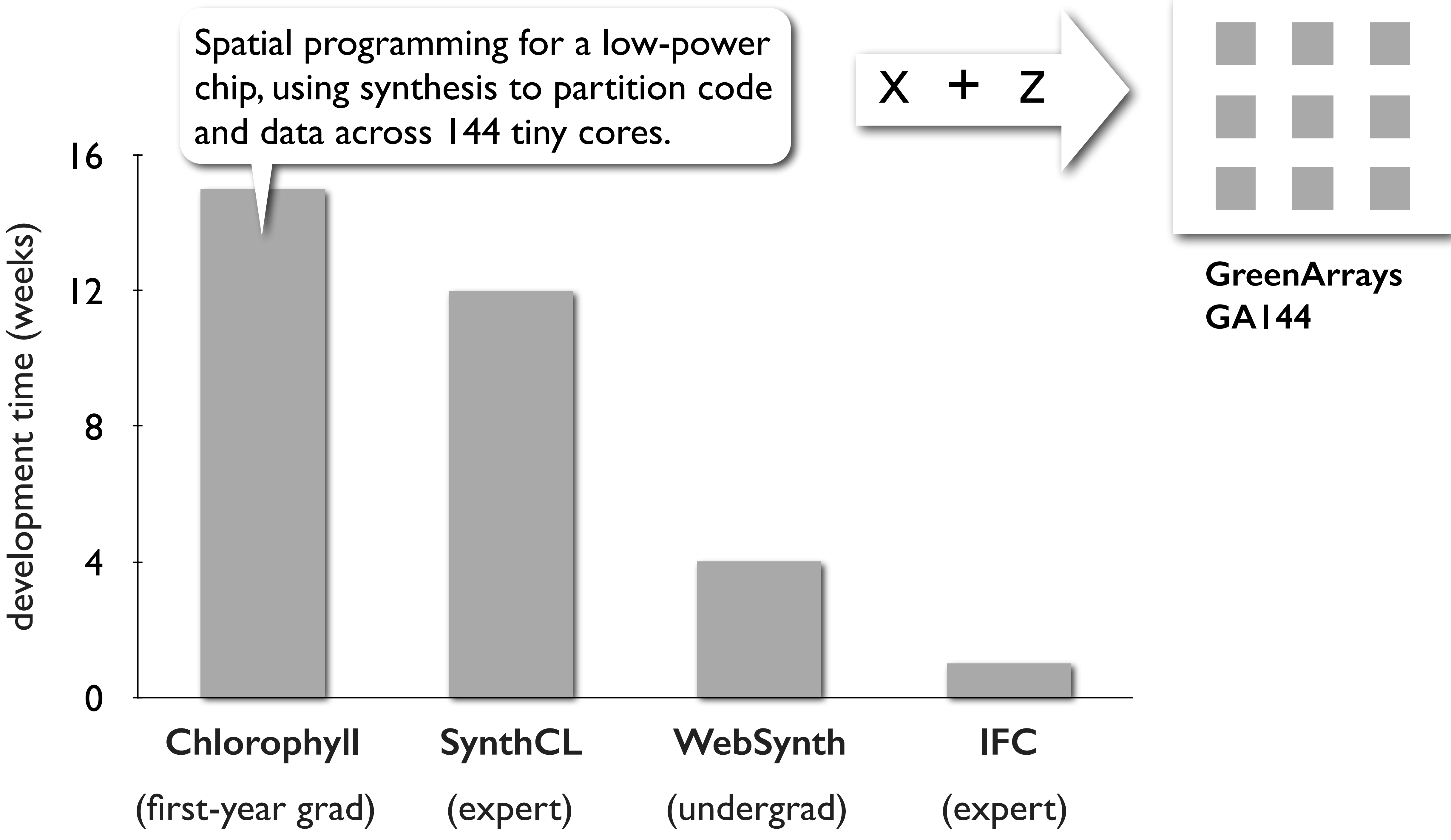
secure stack machines
IFC

[Torlak & Bodik, **Onward'13, PLDI'14**]

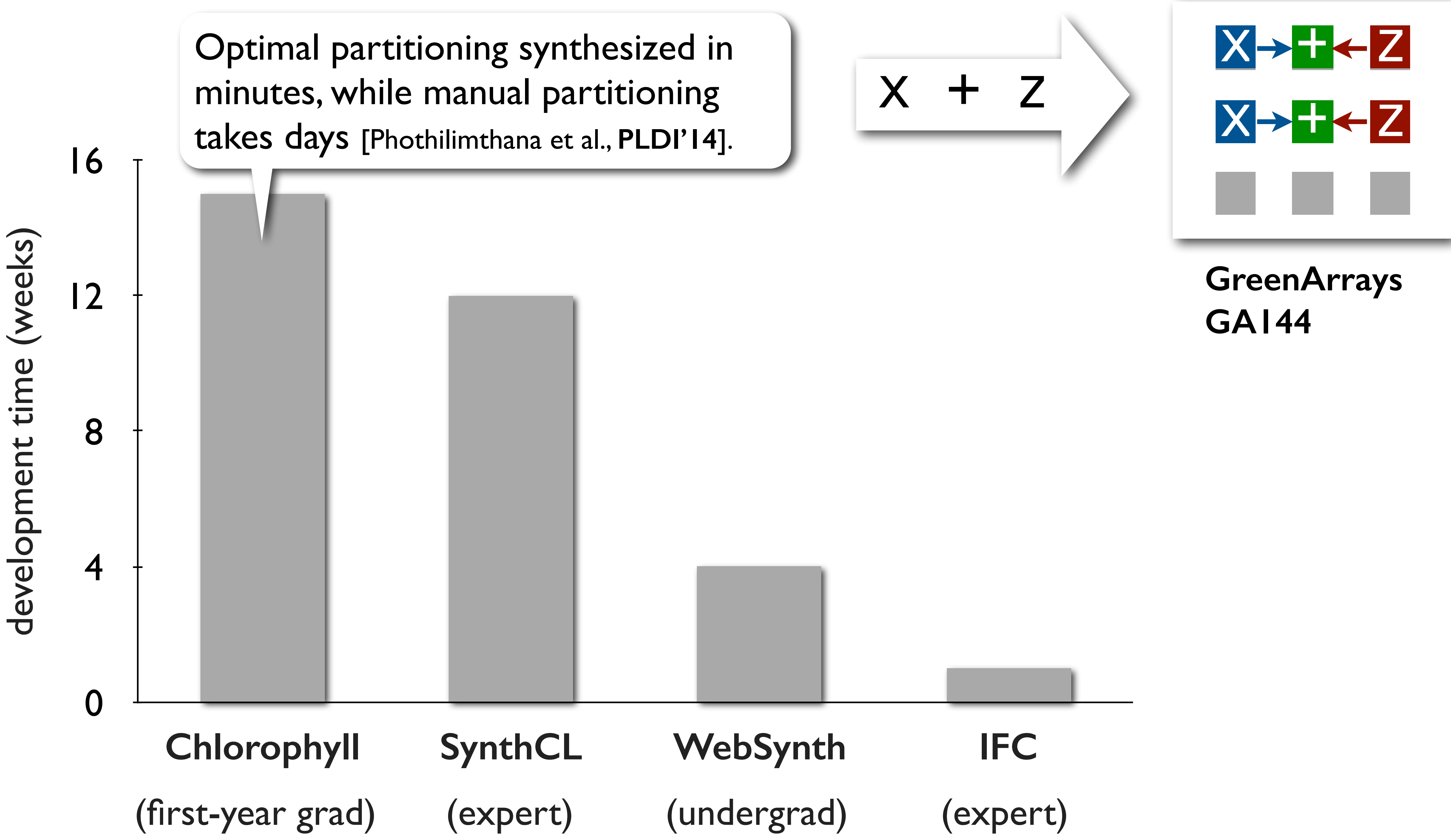
SDSLs developed with ROSETTE



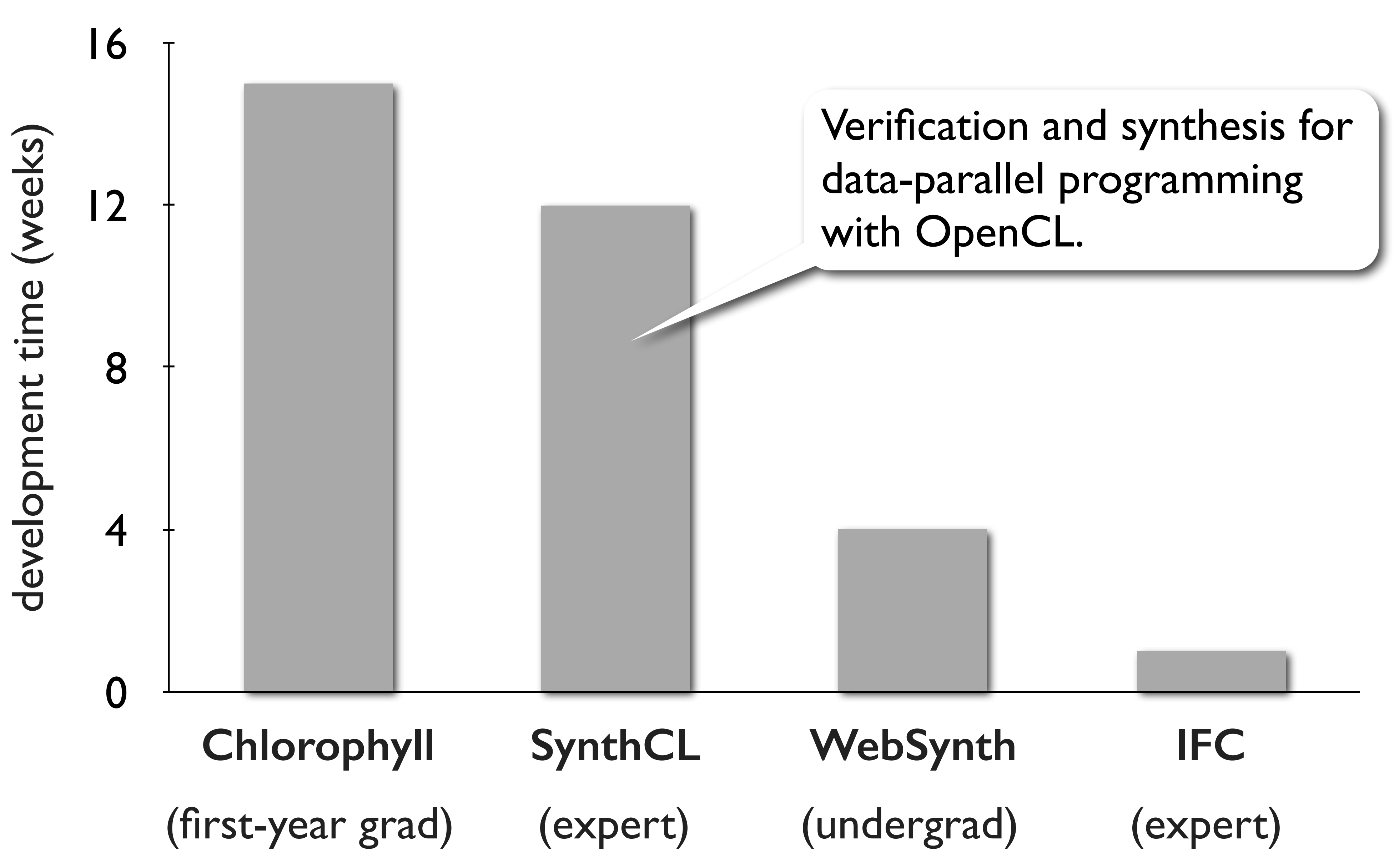
SDSLs developed with ROSETTE



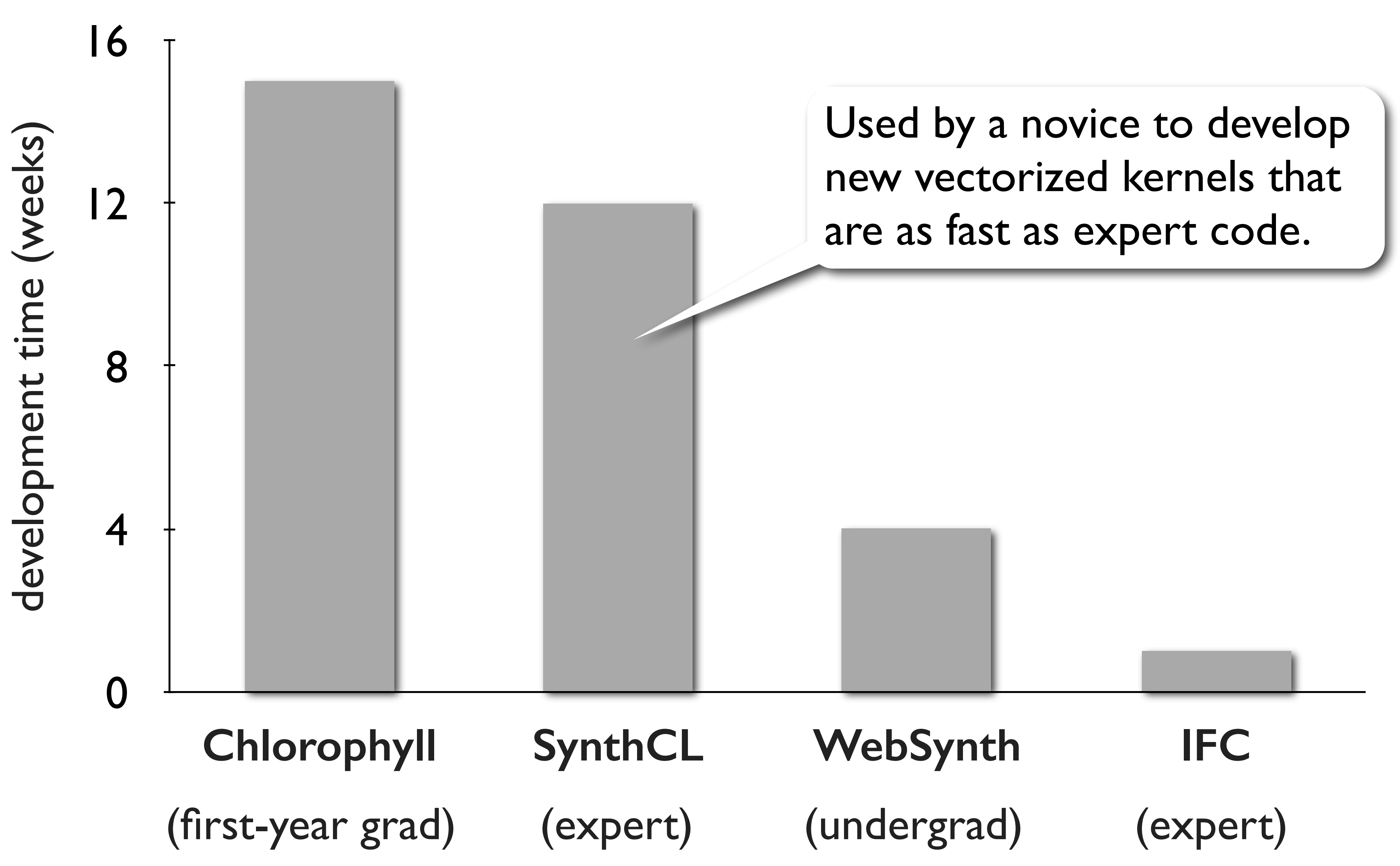
SDSLs developed with ROSETTE



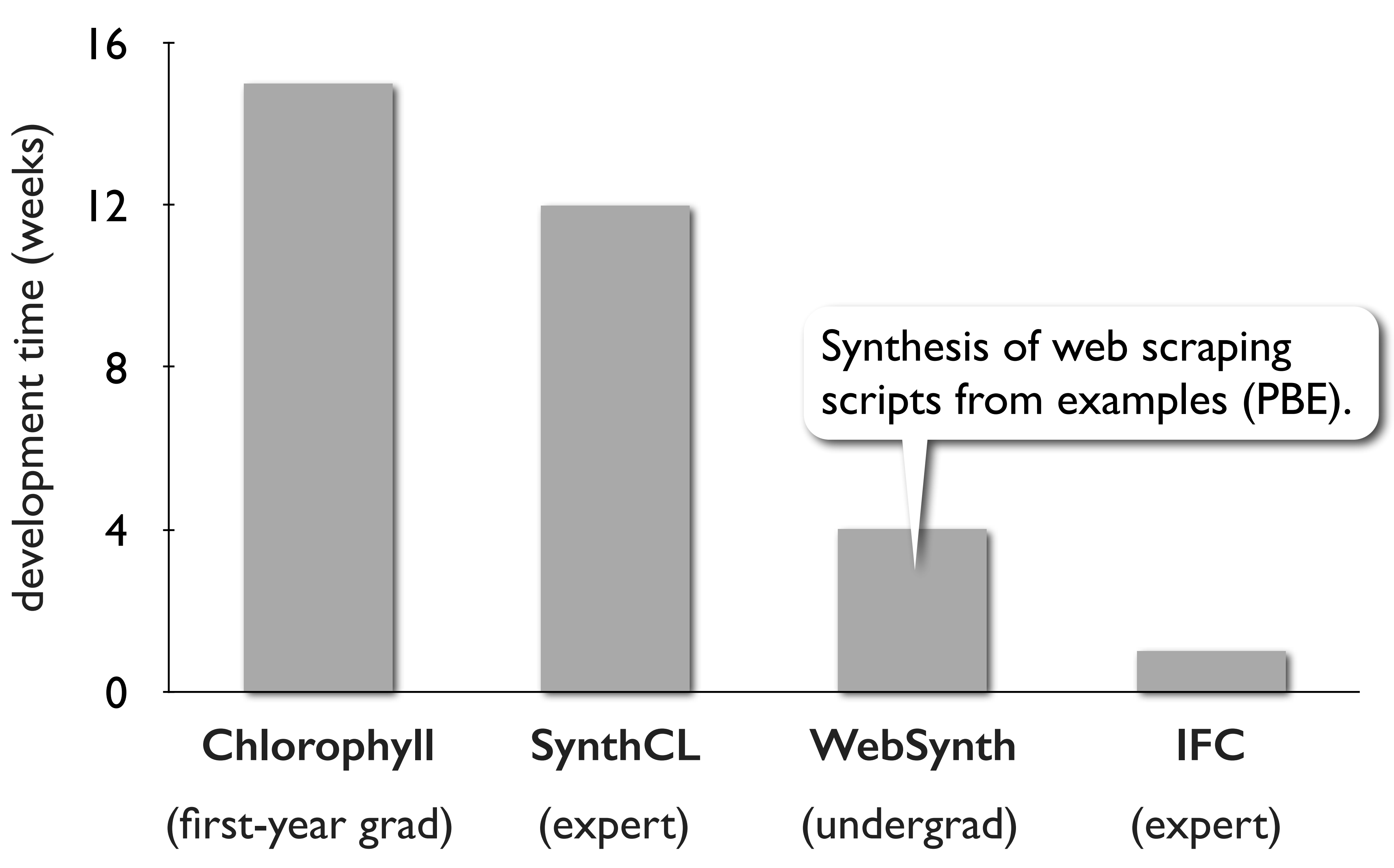
SDSLs developed with ROSETTE



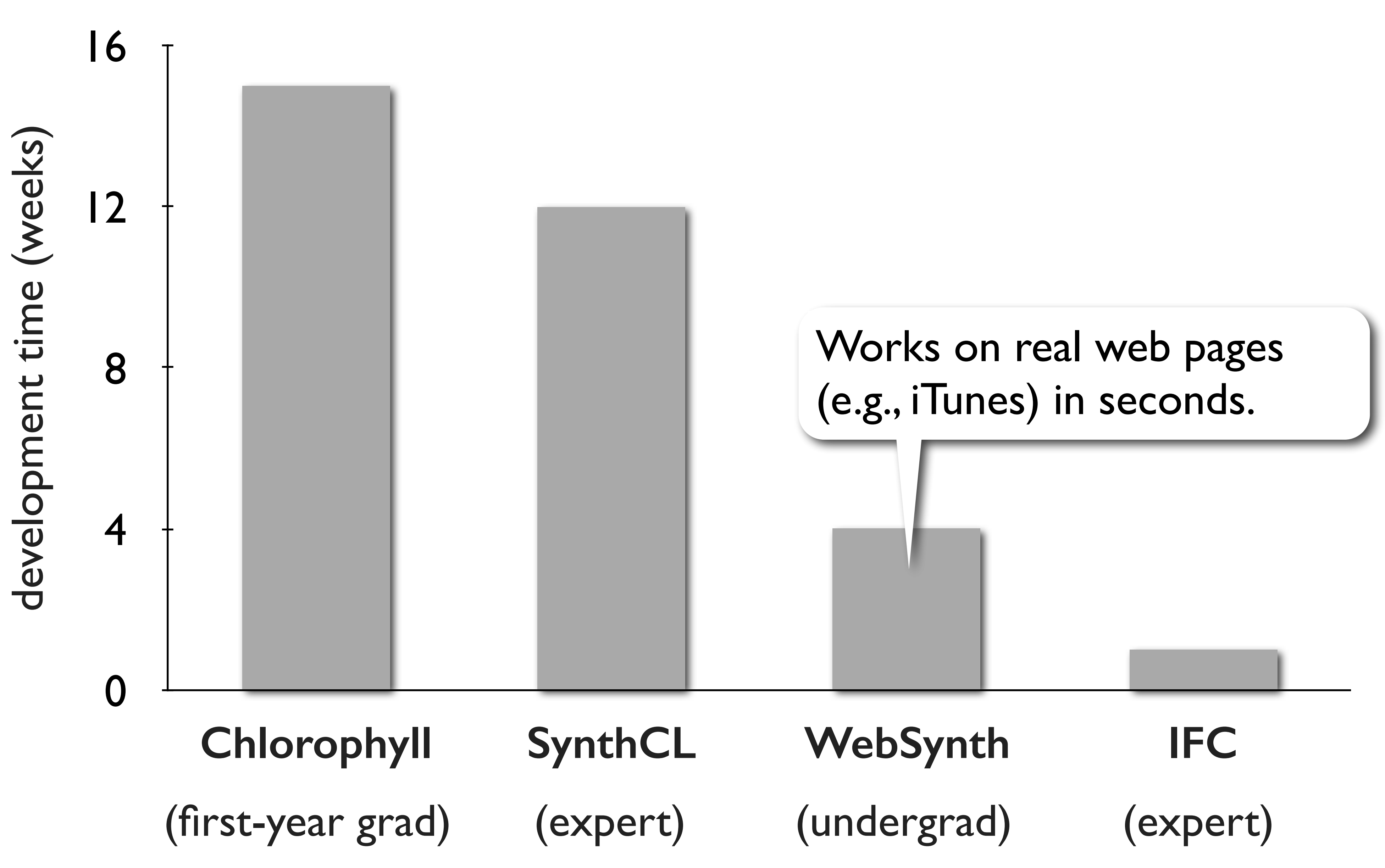
SDSLs developed with ROSETTE



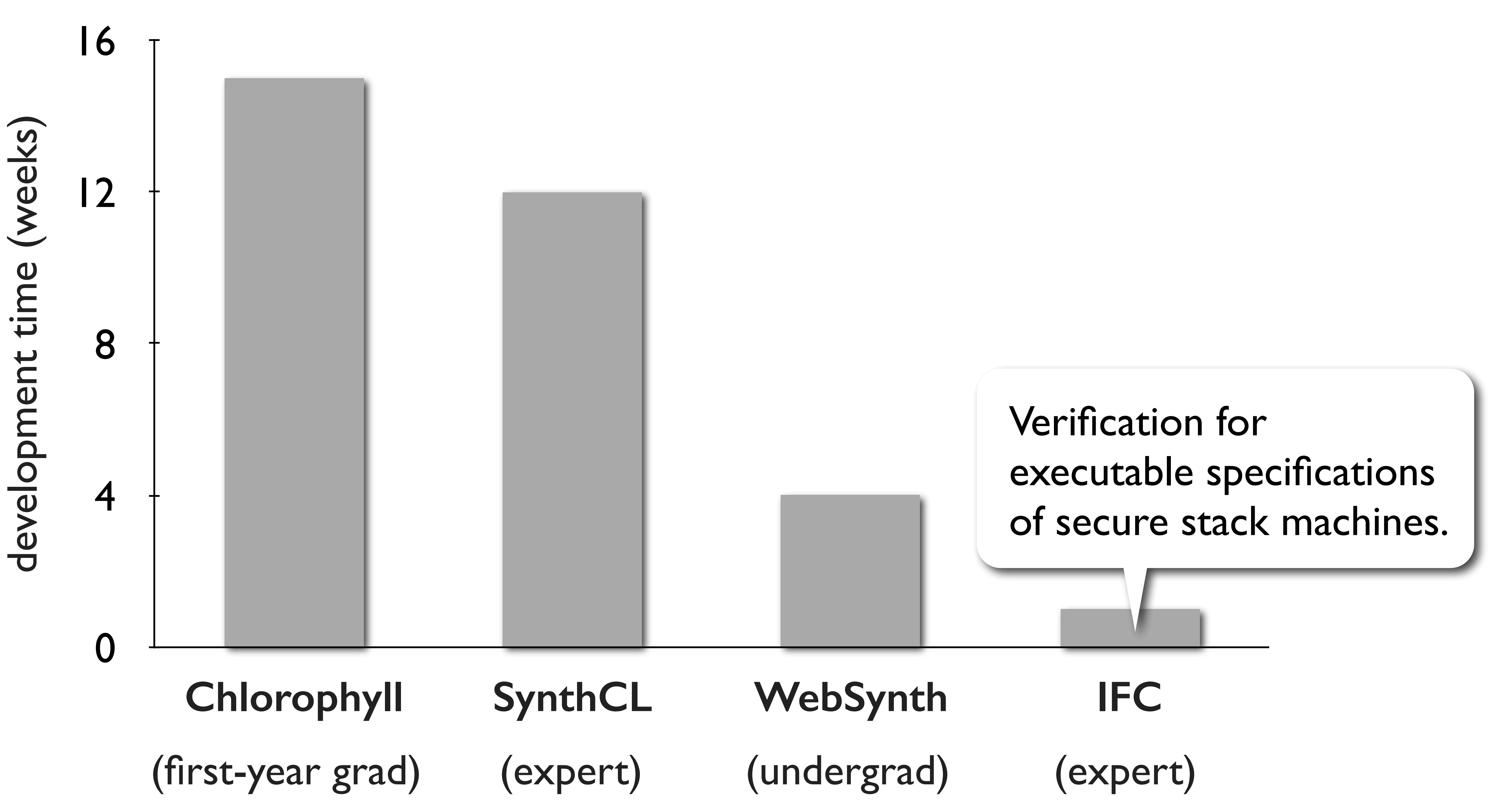
SDSLs developed with ROSETTE



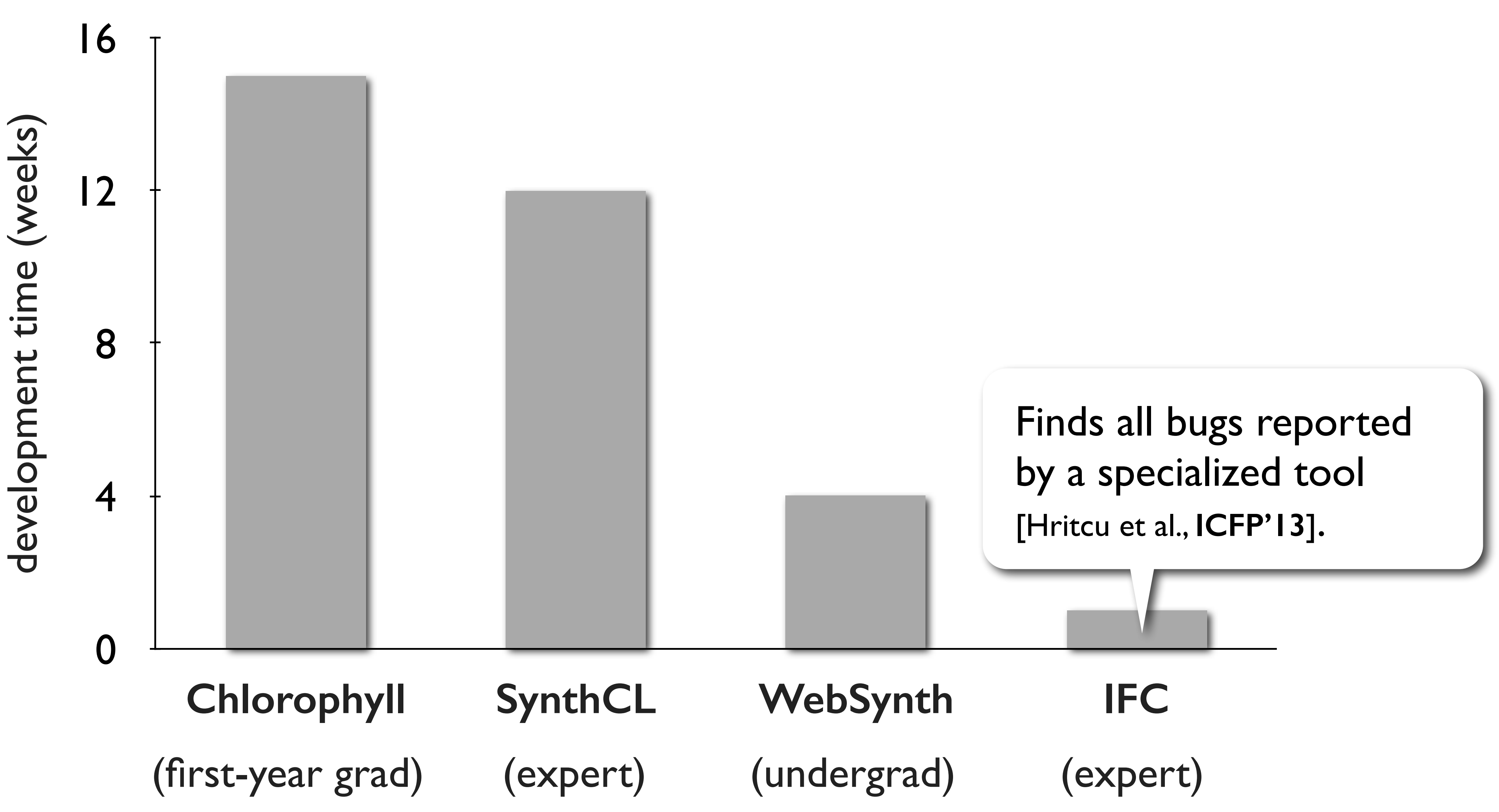
SDSLs developed with ROSETTE



SDSLs developed with ROSETTE



SDSLs developed with ROSETTE



Anatomy of a solver-aided host language

Modern descendent of Scheme with macro-based metaprogramming.

Racket

Anatomy of a solver-aided host language

```
(define-symbolic id type)

(assert expr)

(verify expr)
(debug [expr] expr)
(solve expr)
(synthesize [expr] expr)
```

Anatomy of a solver-aided host language

symbolic constants

```
(define-symbolic id type)

(assert expr)

(verify expr)
(debug [expr] expr)
(solve expr)
(synthesize [expr] expr)
```

ROSETTE

Anatomy of a solver-aided host language

symbolic constants

assertions

```
(define-symbolic id type)
(assert expr)
(verify expr)
(debug [expr] expr)
(solve expr)
(synthesize [expr] expr)
```

ROSETTE

Anatomy of a solver-aided host language

symbolic constants

```
(define-symbolic id type)
```

assertions

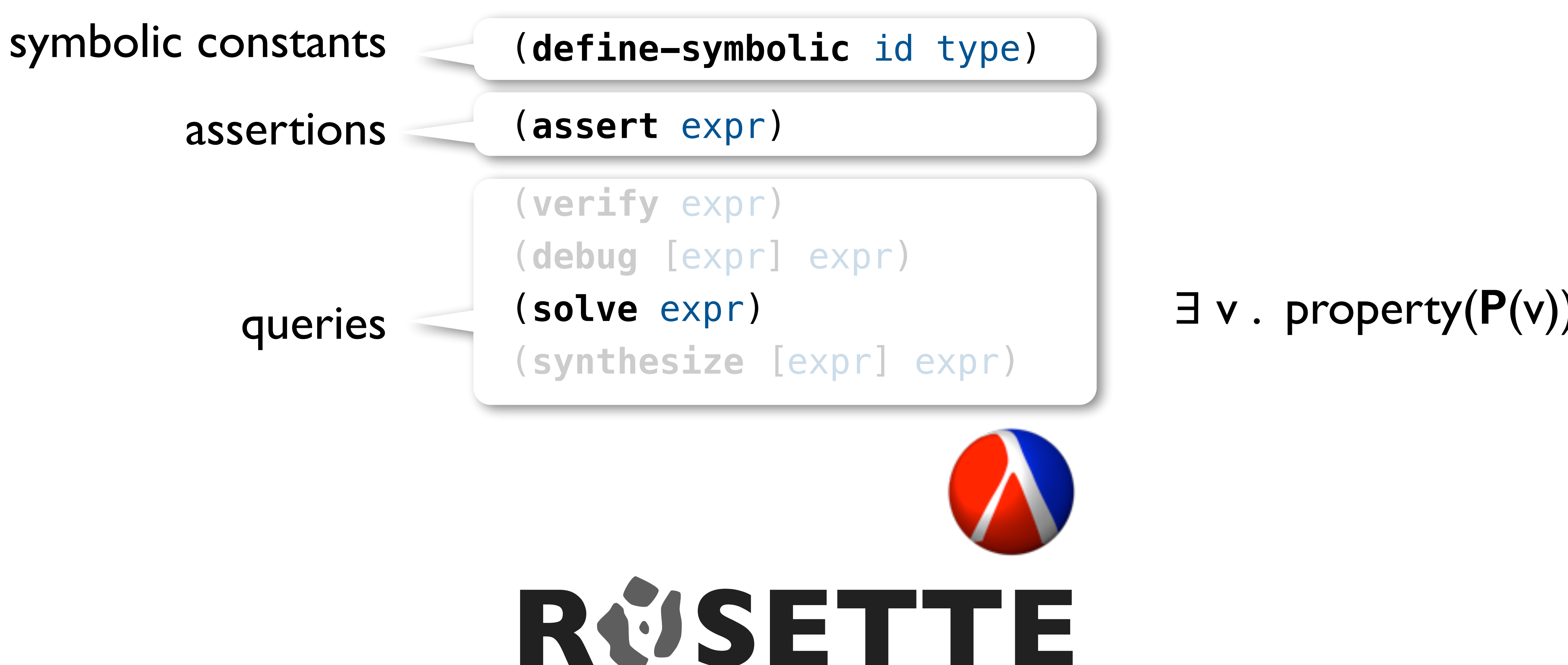
```
(assert expr)
```

queries

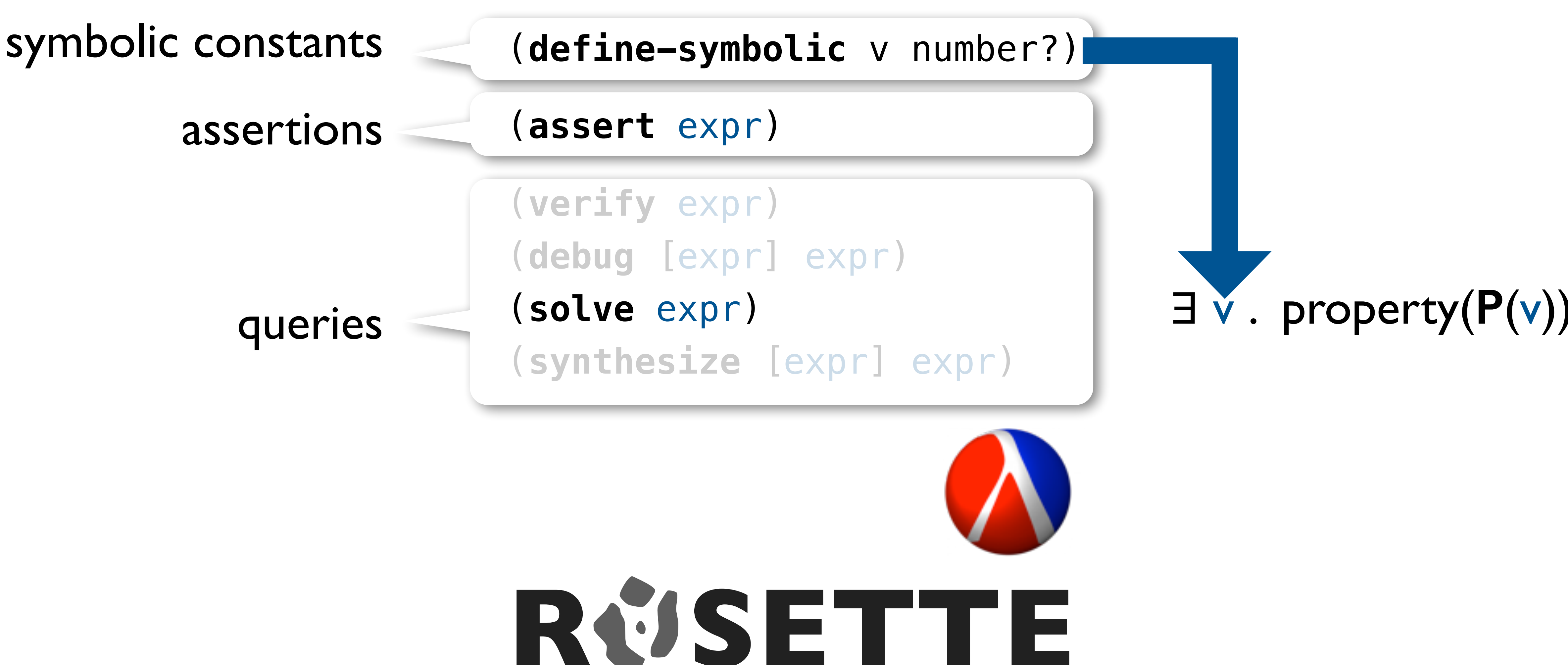
```
(verify expr)
(debug [expr] expr)
(solve expr)
(synthesize [expr] expr)
```

ROSETTE

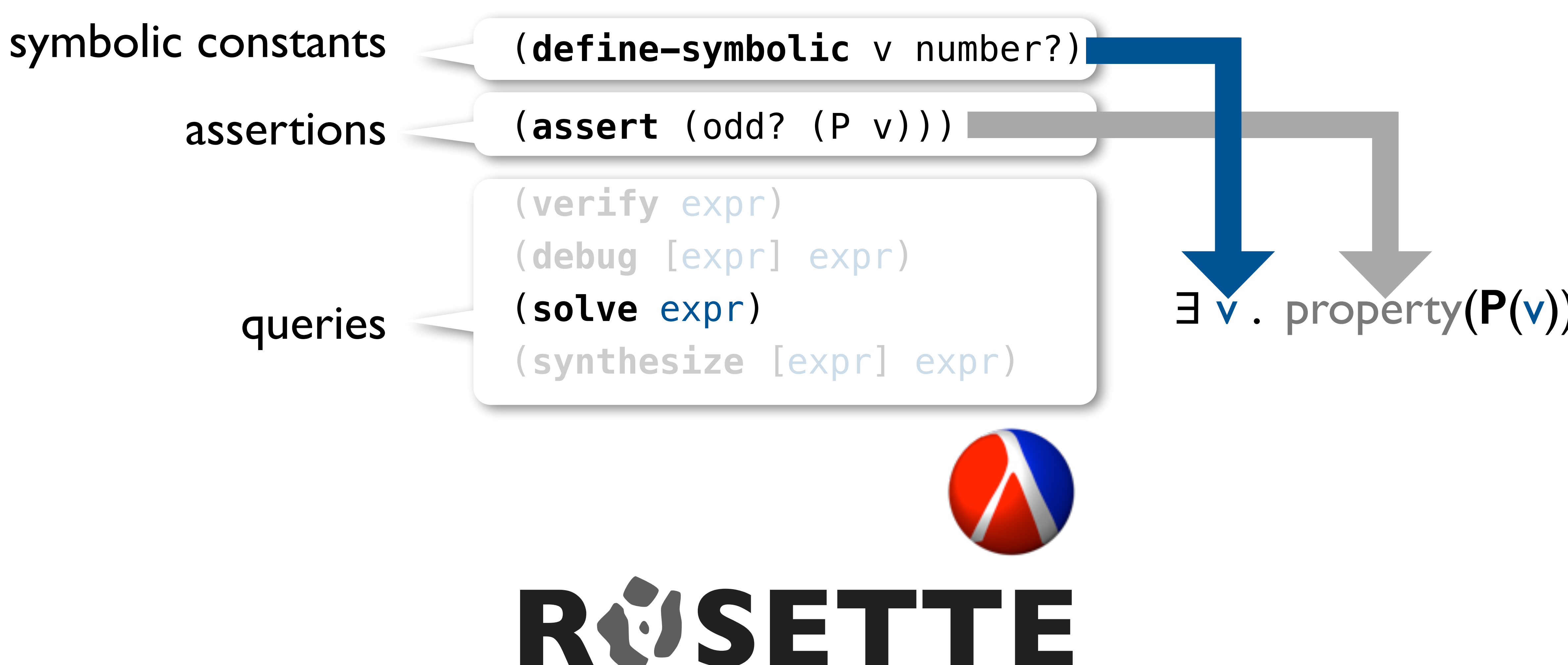
Anatomy of a solver-aided host language



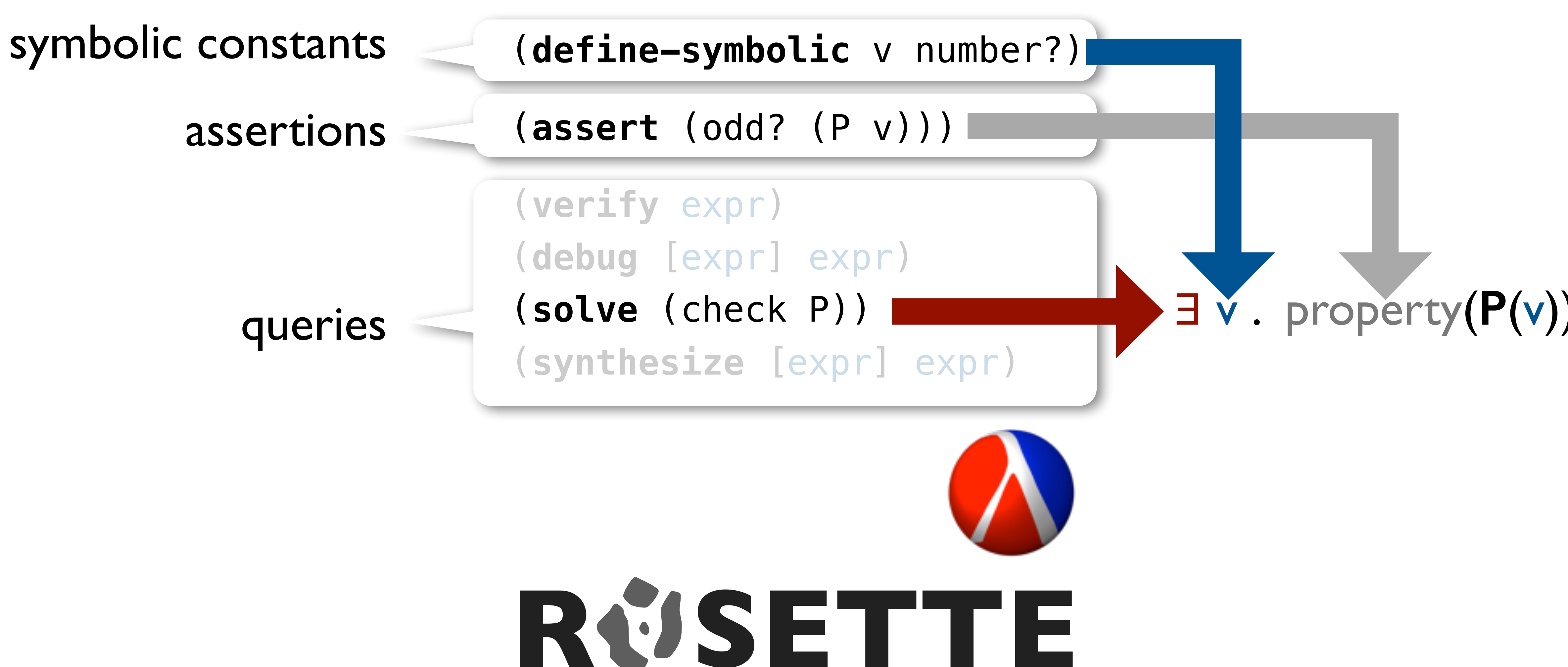
Anatomy of a solver-aided host language



Anatomy of a solver-aided host language



Anatomy of a solver-aided host language



A tiny example SDSL

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6
```

BV: A tiny assembly-like language for writing fast, low-level library functions.

A tiny example SDSL

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6
```

test debug
verify synth

BV: A tiny assembly-like language for writing fast, low-level library functions.

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

parse

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

parse

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

```
(out opcode in ...)
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

interpret

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

```
`(-2 -1)
```

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6
```

```
> bvmax(-2, -1)
```

interpret

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

0	-2
1	-1
2	
3	
4	
5	
6	

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

interpret

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

0	-2
1	-1
2	
3	
4	
5	
6	

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

interpret

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

0	-2
1	-1
2	
3	
4	
5	
6	

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

interpret

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

0	-2
1	-1
2	
3	
4	
5	
6	

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

interpret

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

0	-2
1	-1
2	0
3	
4	
5	
6	

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
```

interpret

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

0	-2
1	-1
2	0
3	0
4	-2
5	0
6	-1

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
-1
```

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

0	-2
1	-1
2	0
3	0
4	-2
5	0
6	-1

interpret

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> bvmax(-2, -1)
-1
```

```
(define bvmax
 `((2 bvge 0 1)
   (3 bvneg 2)
   (4 bvxor 0 2)
   (5 bvand 3 4)
   (6 bvxor 1 5)))
```

- pattern matching
- dynamic evaluation
- first-class & higher-order procedures
- side effects

```
(define (interpret prog inputs)
  (make-registers prog inputs)
  (for ([stmt prog])
    (match stmt
      [(list out opcode in ...)
       (define op (eval opcode))
       (define args (map load in))
       (store out (apply op args))]))
  (load (last)))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> verify(bvmax, max)
```

query

```
(define-symbolic n0 n1 number?)
(define inputs (list n0 n1))
(verify
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> verify(bvmax, max)
```

query

Creates two fresh symbolic constants of type number and binds them to variables n0 and n1.

```
(define-symbolic n0 n1 number?)
(define inputs (list n0 n1))
(verify
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> verify(bvmax, max)
```

query

Symbolic values can be used just like concrete values of the same type.

```
(define-symbolic n0 n1 number?)
(define inputs (list n0 n1))
(verify
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> verify(bvmax, max)
(0, -2)
```

query

```
(define-symbolic n0 n1 number?)
(define inputs (list n0 n1))
(verify
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

(verify *expr*) searches for a concrete interpretation of symbolic constants that causes *expr* to fail.

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> verify(bvmax, max)
(0, -2)

> bvmax(0, -2)
-1
```

query

```
(define-symbolic n0 n1 number?)
(define inputs (list n0 n1))
(verify
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> debug(bvmax, max, (0, -2))
```

query

```
(define inputs (list 0 -2))
(debug [input-register?]
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r2)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6
```

```
> debug(bvmax, max, (0, -2))
```

query

```
(define inputs (list 0 -2))
(debug [input-register?]
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(??, ??)
  r5 = bvand(r3, ??)
  r6 = bvxor(??, ??)
  return r6

> synthesize(bvmax, max)
```

query

```
(define-symbolic n0 n1 number?)
(define inputs (list n0 n1))
(synthesize [inputs]
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

A tiny example SDSL: ROSETTE

```
def bvmax(r0, r1) :
  r2 = bvge(r0, r1)
  r3 = bvneg(r2)
  r4 = bvxor(r0, r1)
  r5 = bvand(r3, r4)
  r6 = bvxor(r1, r5)
  return r6

> synthesize(bvmax, max)
```

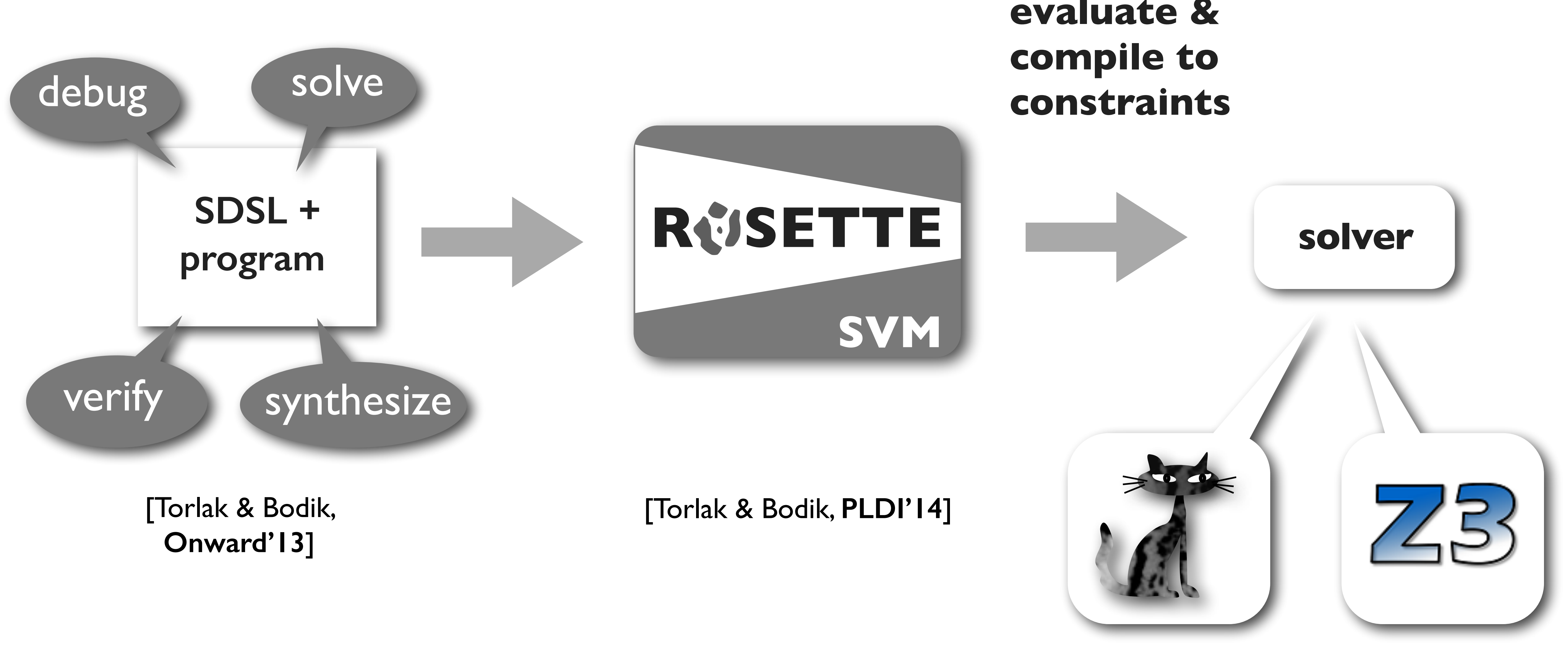
query

```
(define-symbolic n0 n1 number?)
(define inputs (list n0 n1))
(synthesize [inputs]
  (assert (= (interpret bvmax inputs)
             (apply max inputs))))
```

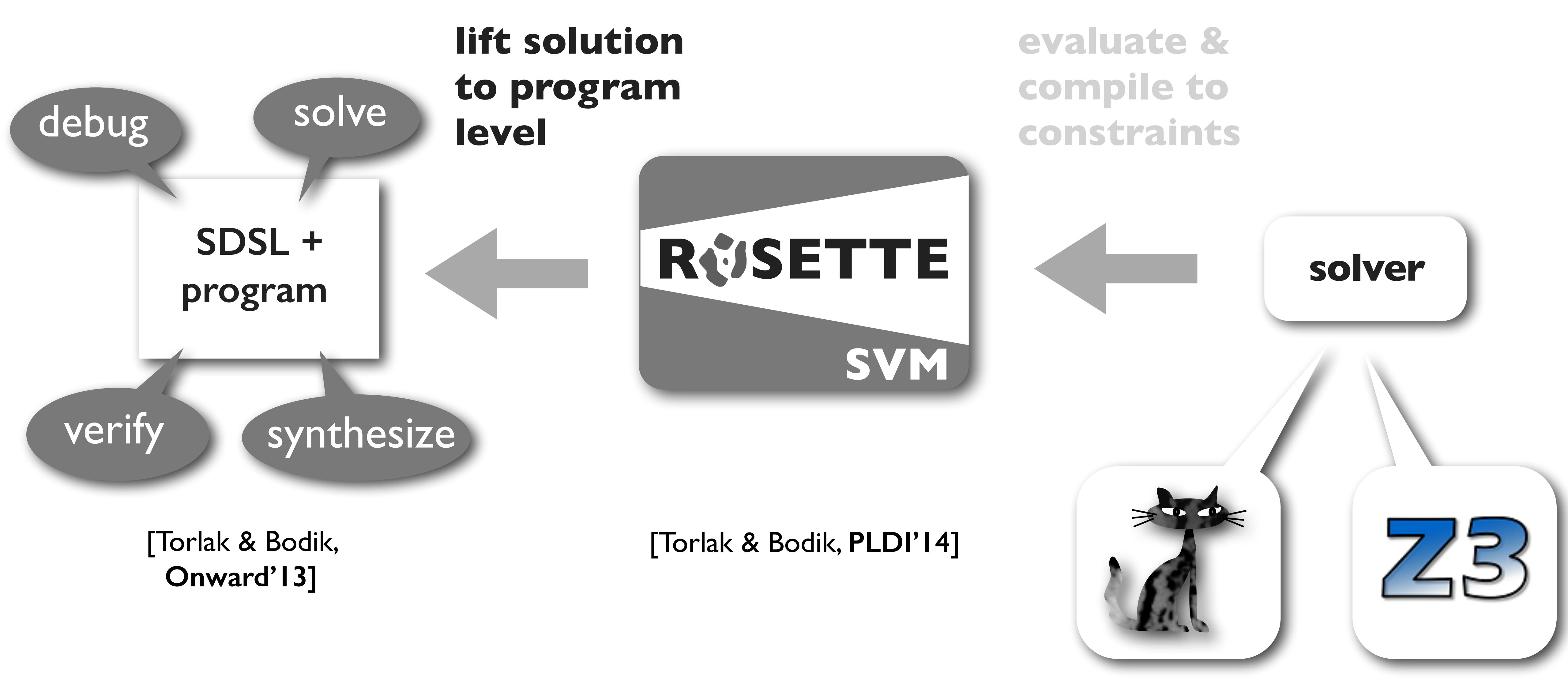
tech

symbolic virtual machine (SVM)

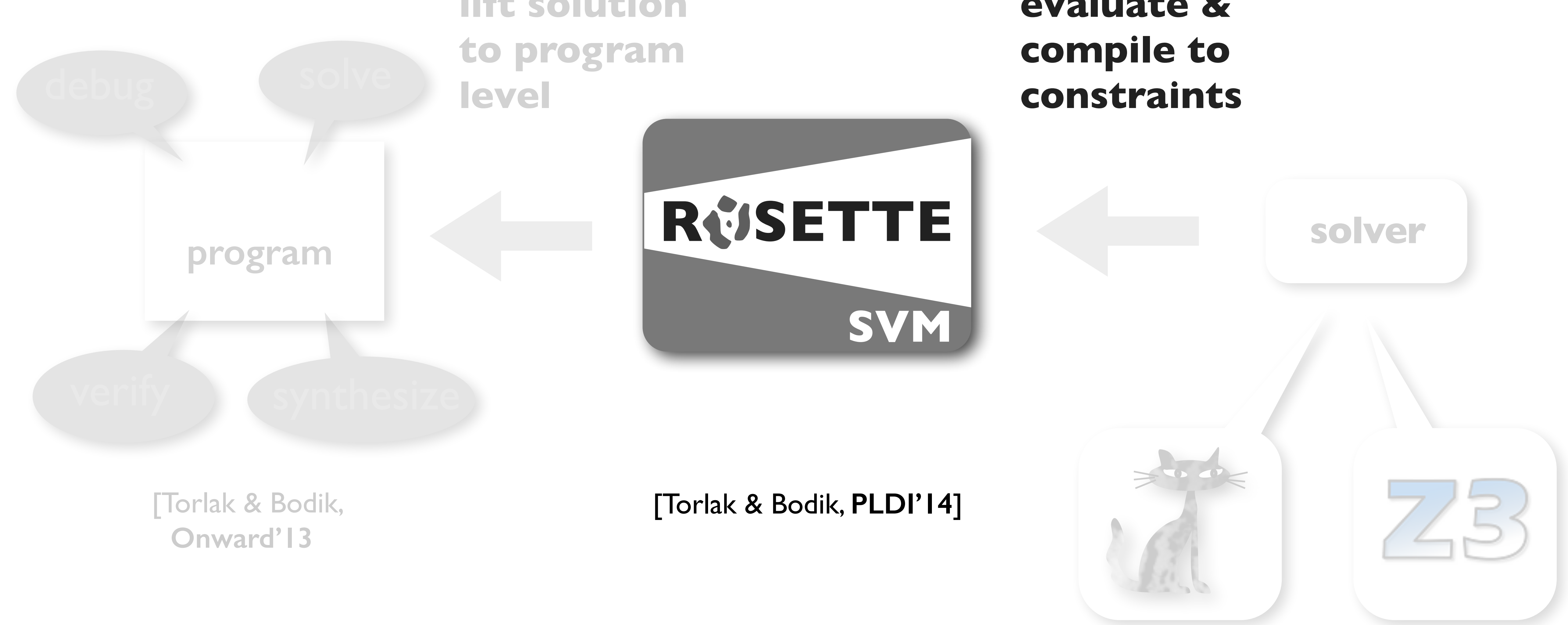
Anatomy of a symbolic virtual machine



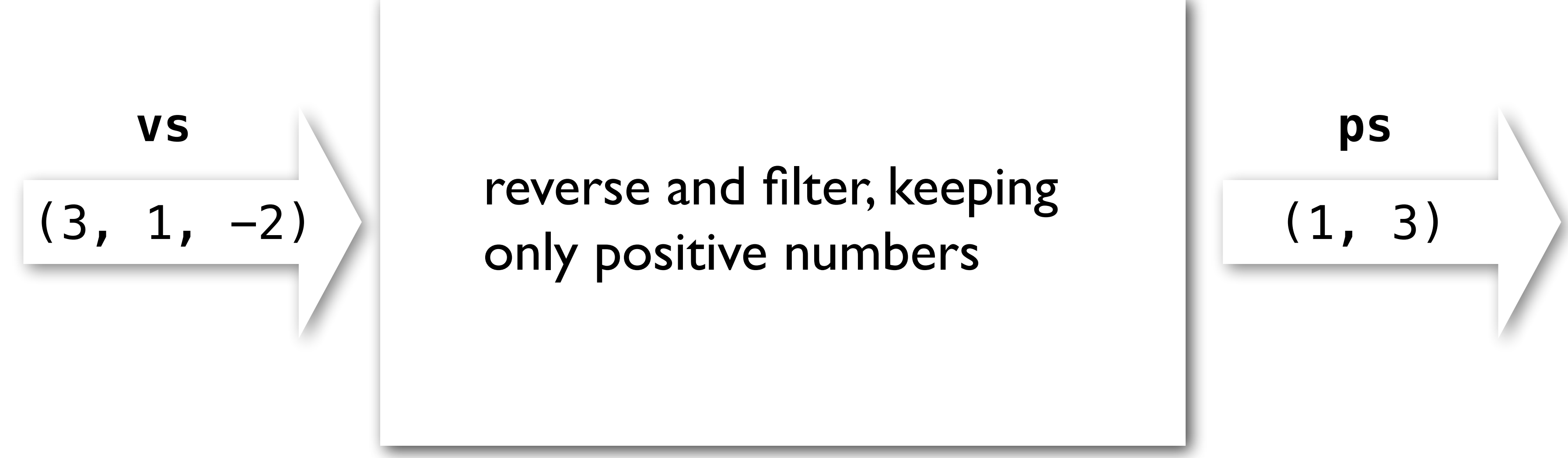
Anatomy of a symbolic virtual machine



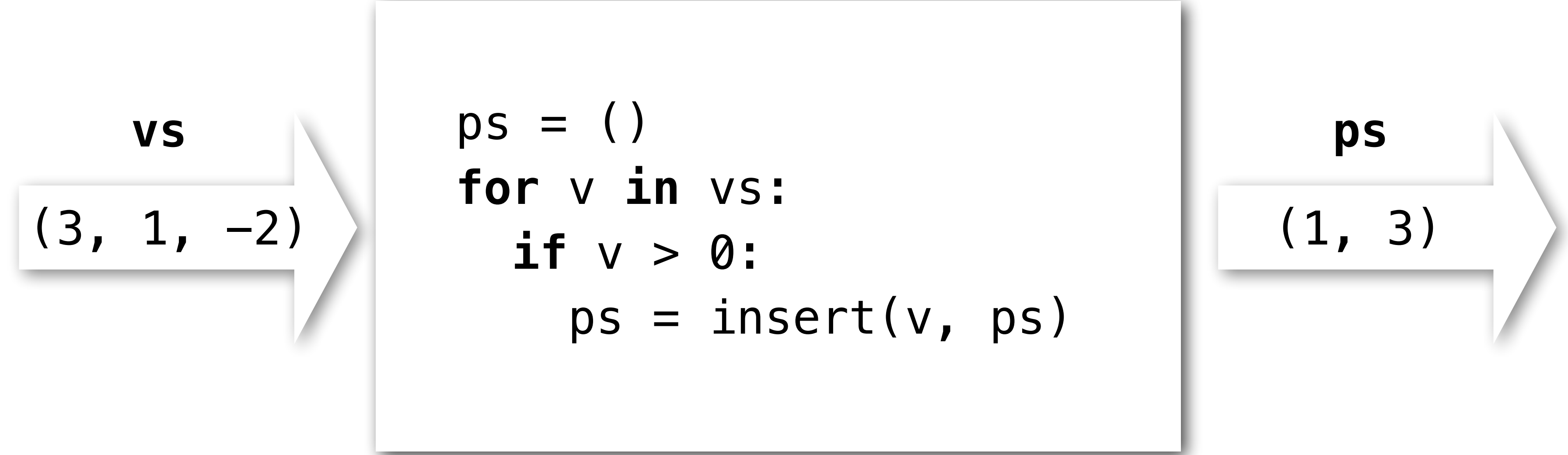
Anatomy of a symbolic virtual machine



Translation to constraints by example



Translation to constraints by example



Translation to constraints by example

vs

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

constraints

Translation to constraints by example

vs

(a, b)

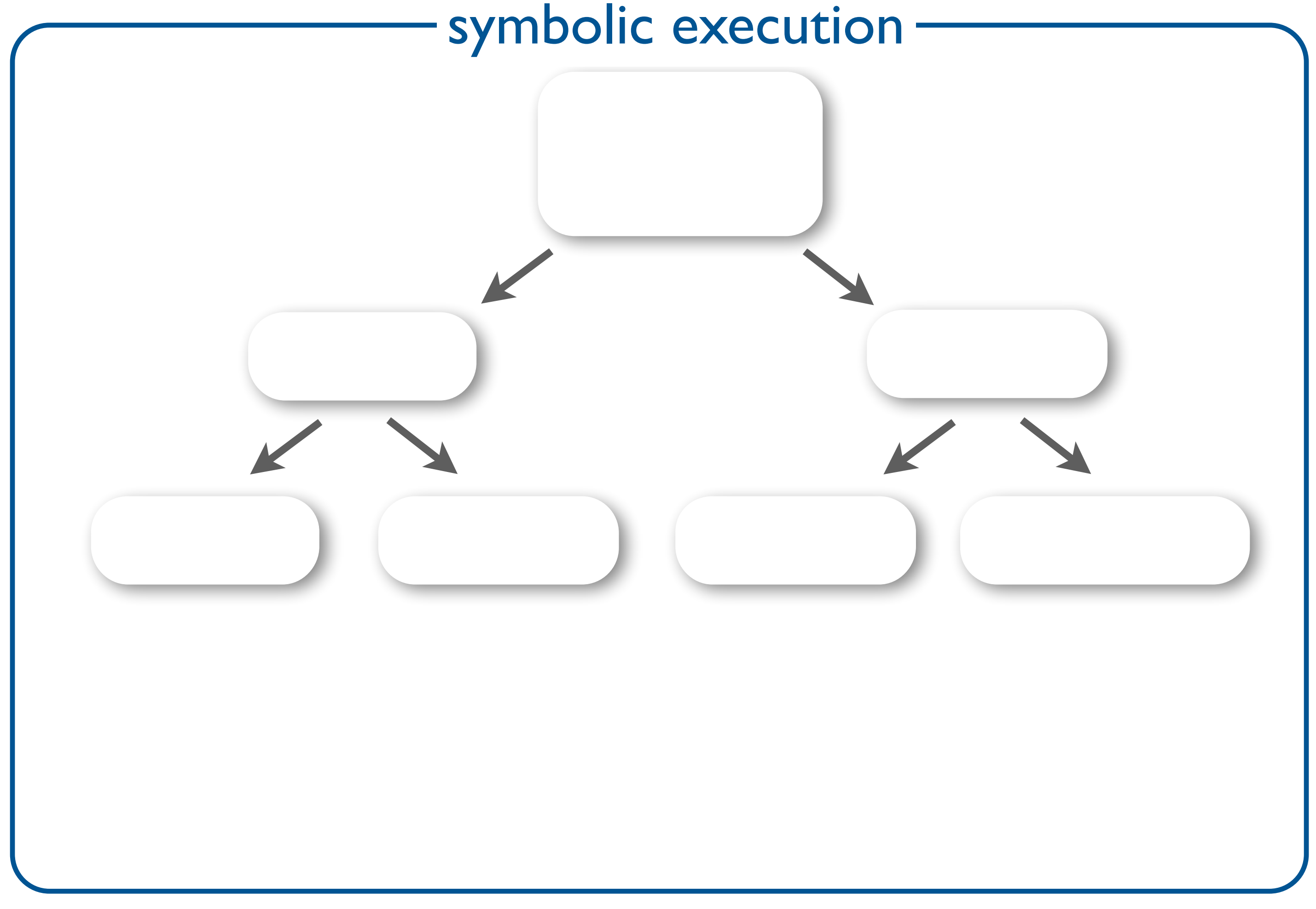
```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

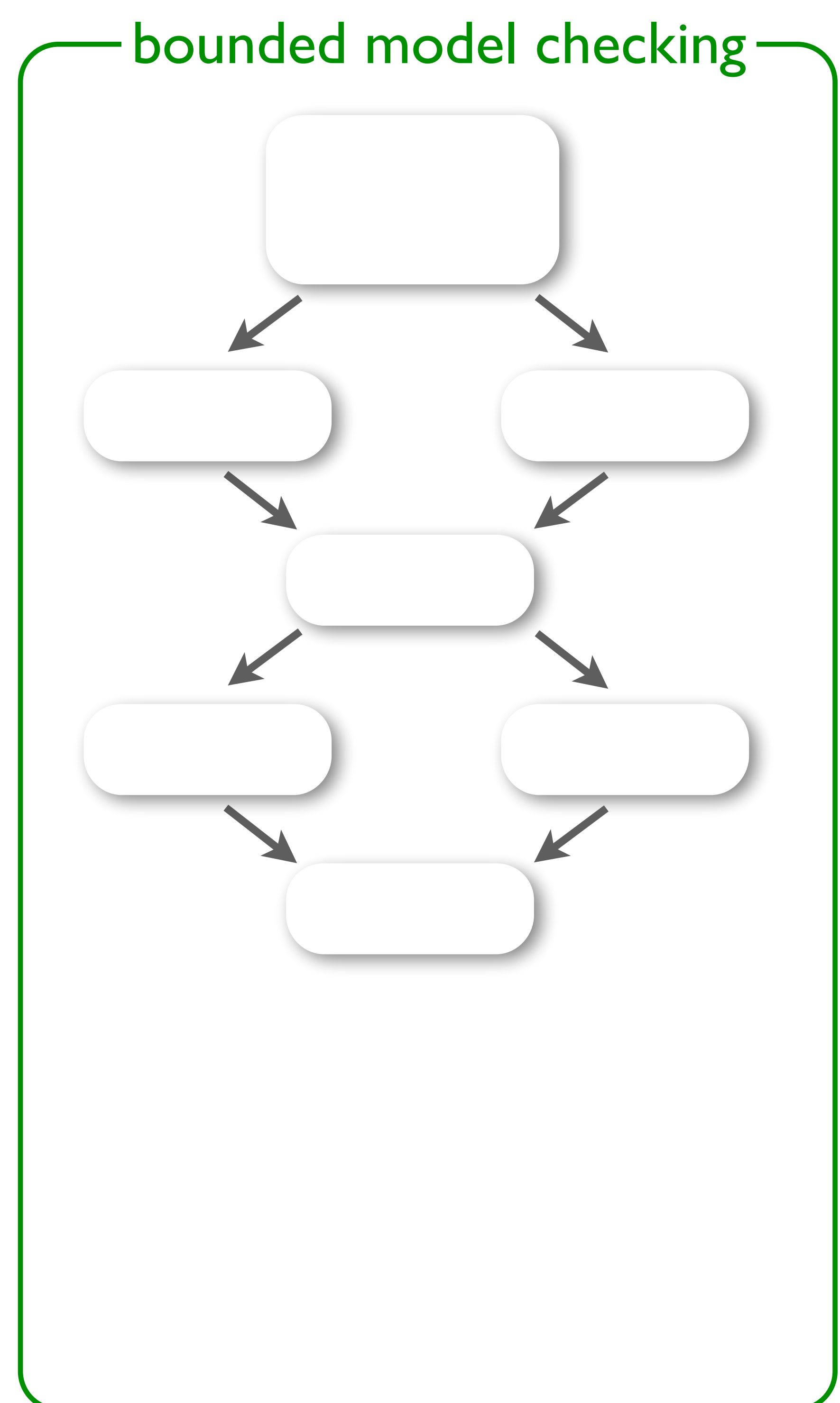
constraints

$a>0 \land b>0$

Design space of precise symbolic encodings

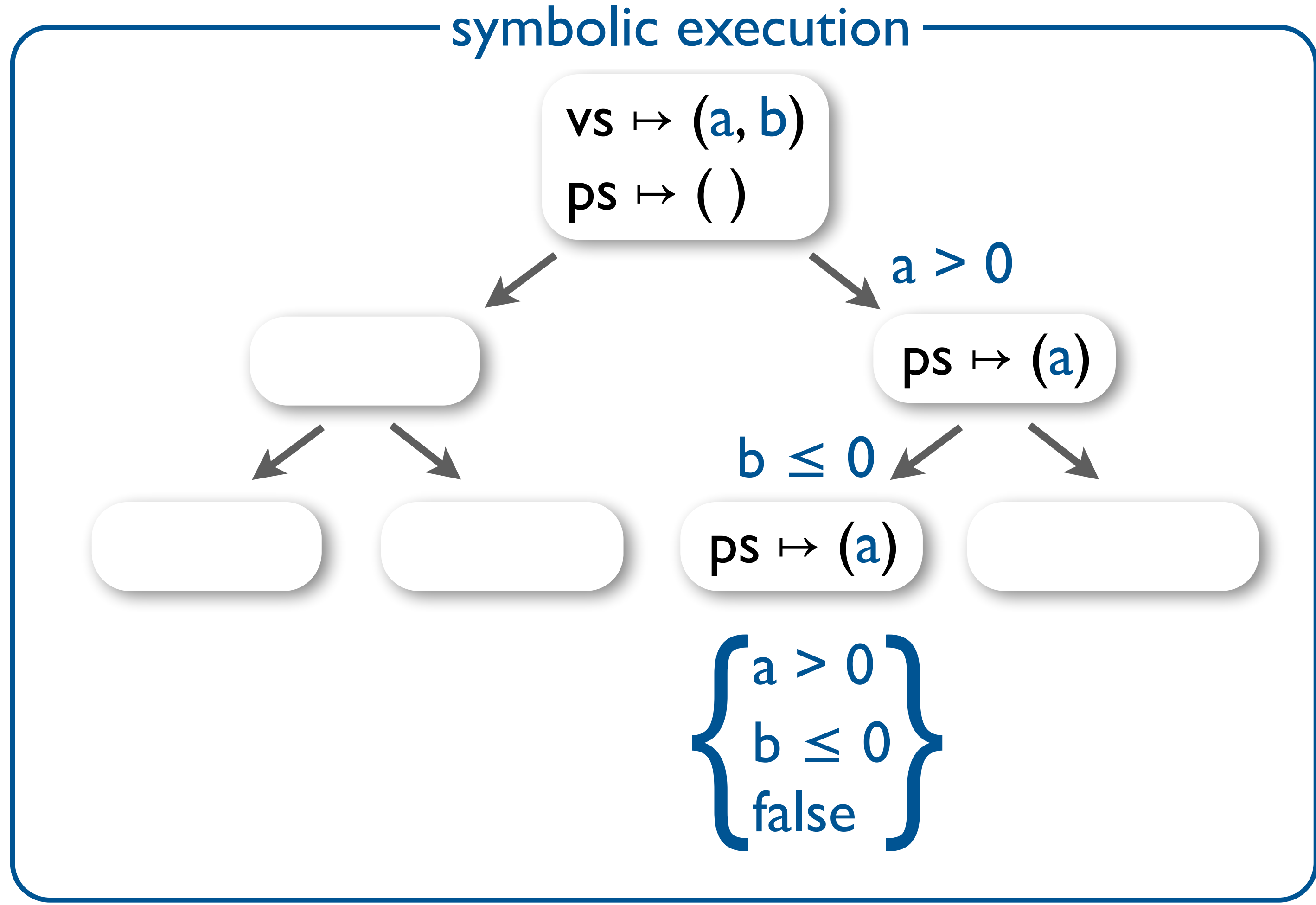
```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

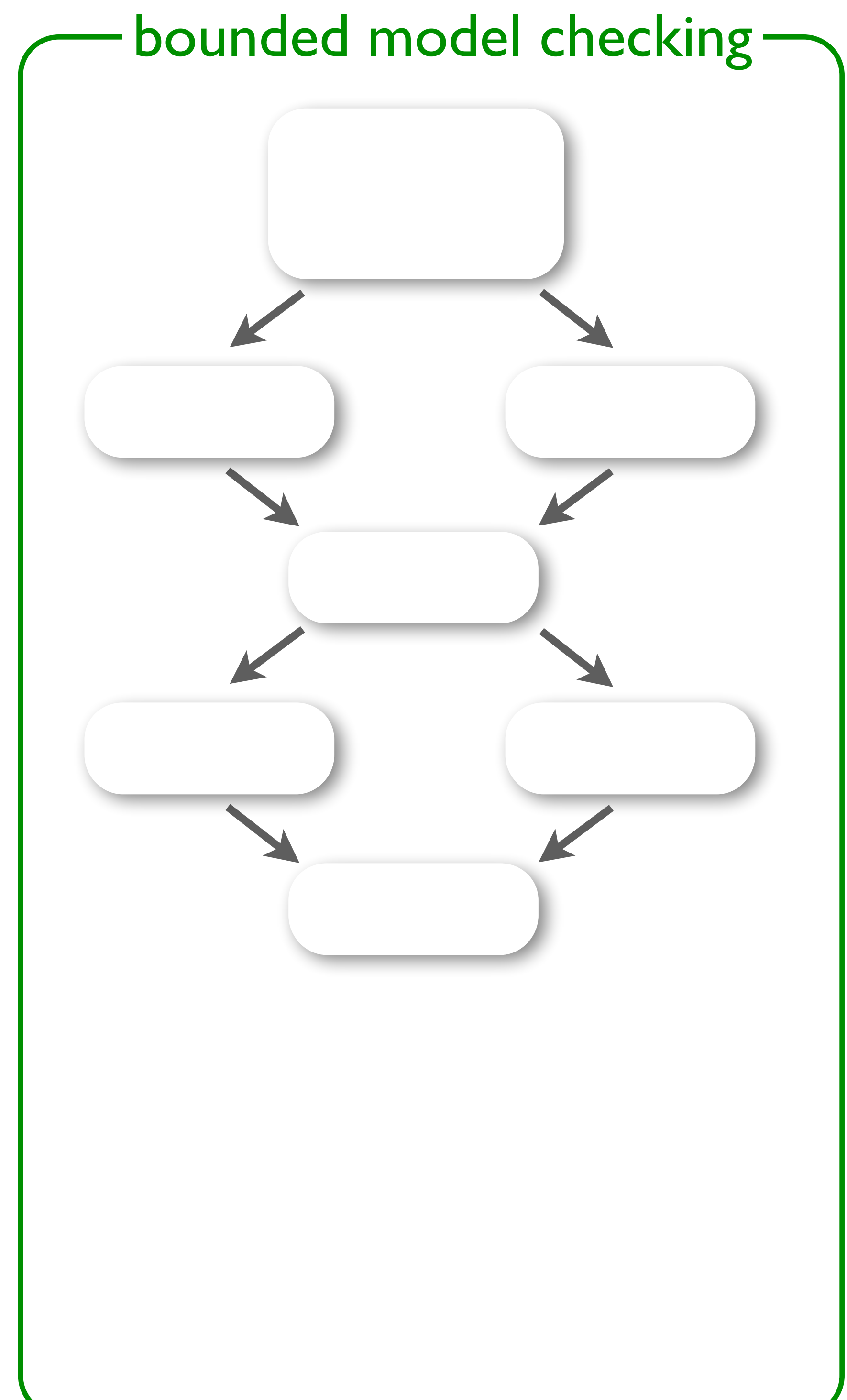




Design space of precise symbolic encodings

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```





Design space of precise symbolic encodings

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

symbolic execution

vs ↦ (a, b)
ps ↦ ( )

$a \leq 0$

$a > 0$

ps ↦ ( )

ps ↦ (a)

$b \leq 0$

$b > 0$

$b \leq 0$

$b > 0$

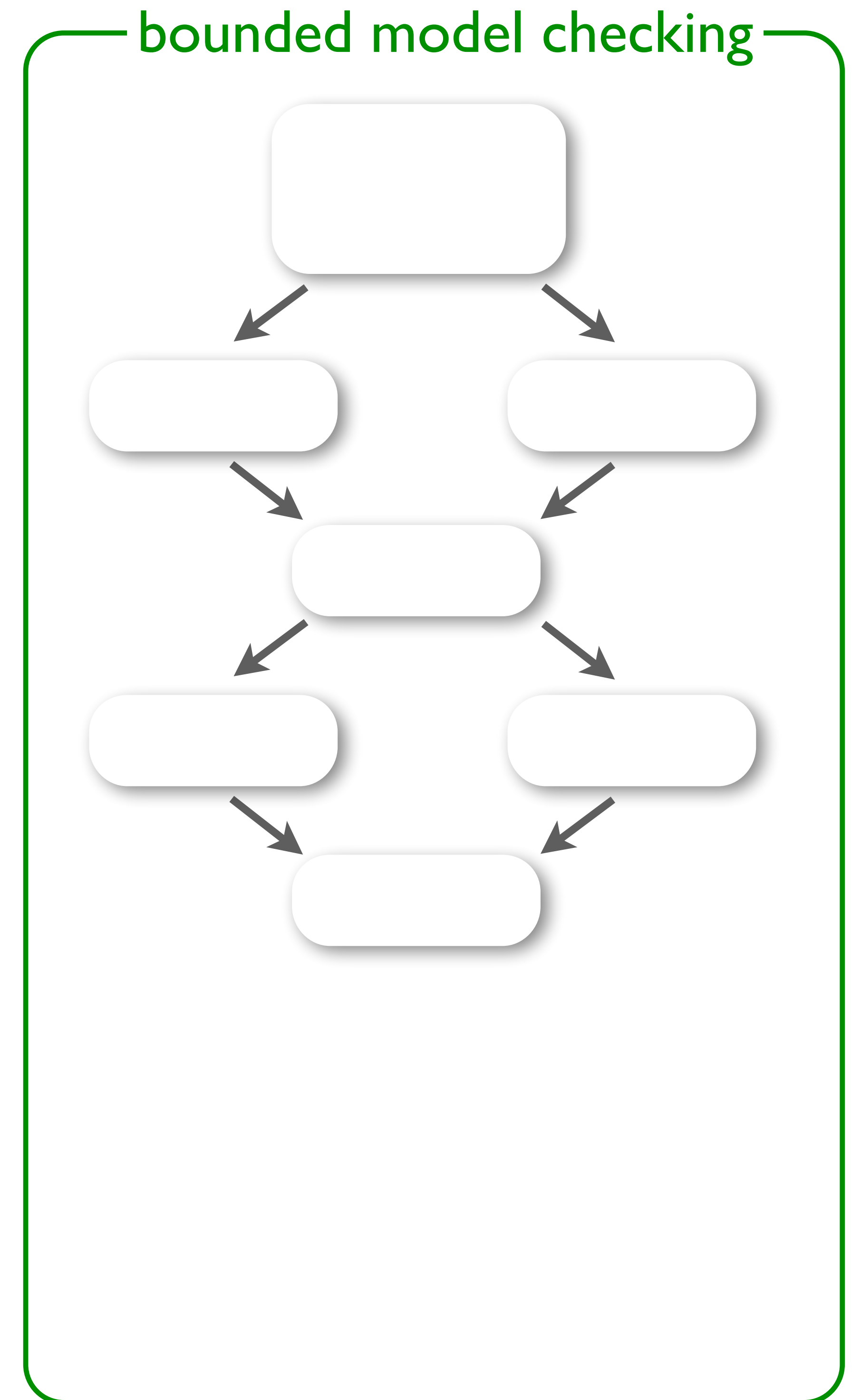
ps ↦ ( )

ps ↦ (b)

ps ↦ (a)

ps ↦ (b, a)

$$\begin{Bmatrix} a \leq 0 \\ b \leq 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a \leq 0 \\ b > 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b \leq 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b > 0 \\ \text{true} \end{Bmatrix}$$



Design space of precise symbolic encodings

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

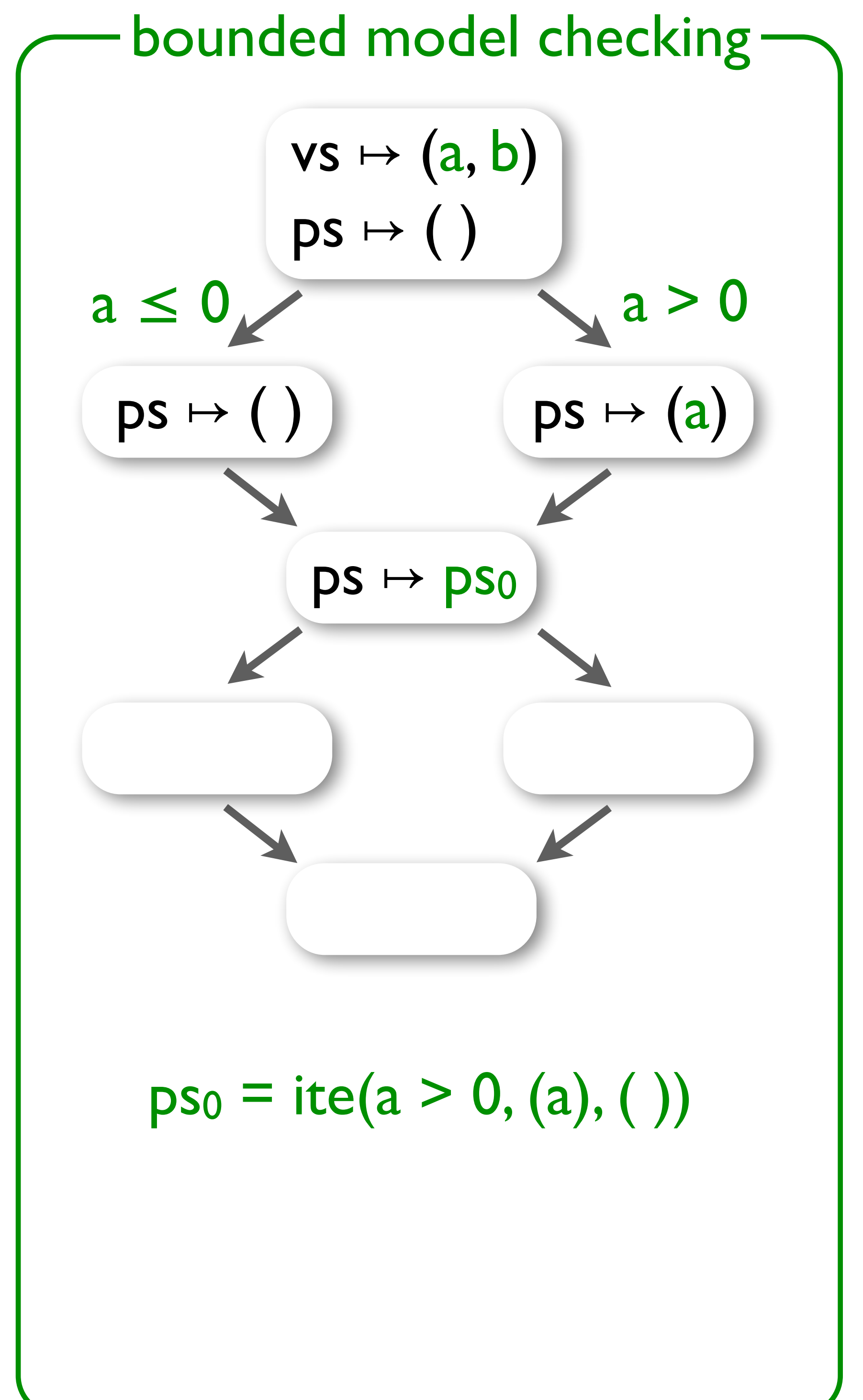
symbolic execution

vs ↦ (a, b)
ps ↦ ( )

$a \le 0$ → ps ↦ ( )
$a > 0$ → ps ↦ (a)

$b \le 0$ → ps ↦ ( )
$b > 0$ → ps ↦ (b)
$b \le 0$ → ps ↦ (a)
$b > 0$ → ps ↦ (b, a)

$$\begin{Bmatrix} a \le 0 \\ b \le 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a \le 0 \\ b > 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b \le 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b > 0 \\ \text{true} \end{Bmatrix}$$



Design space of precise symbolic encodings

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

symbolic execution

$vs \mapsto (a, b)$
$ps \mapsto (\,)$

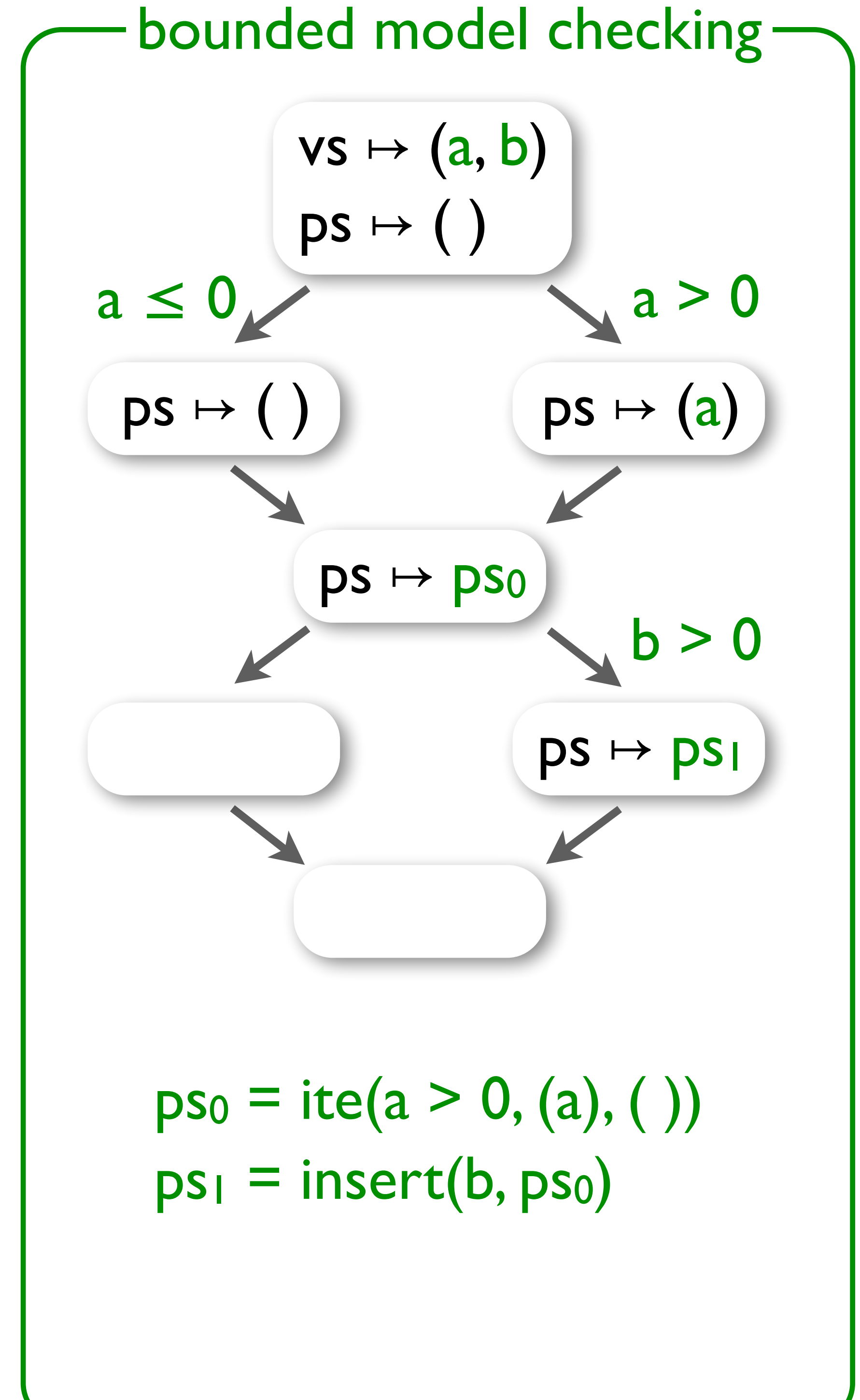
$a \leq 0$ $a > 0$

$ps \mapsto (\,)$ $ps \mapsto (a)$

$b \leq 0$ $b > 0$ $b \leq 0$ $b > 0$

$ps \mapsto (\,)$ $ps \mapsto (b)$ $ps \mapsto (a)$ $ps \mapsto (b, a)$

$$\begin{Bmatrix} a \leq 0 \\ b \leq 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a \leq 0 \\ b > 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b \leq 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b > 0 \\ \text{true} \end{Bmatrix}$$



Design space of precise symbolic encodings

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

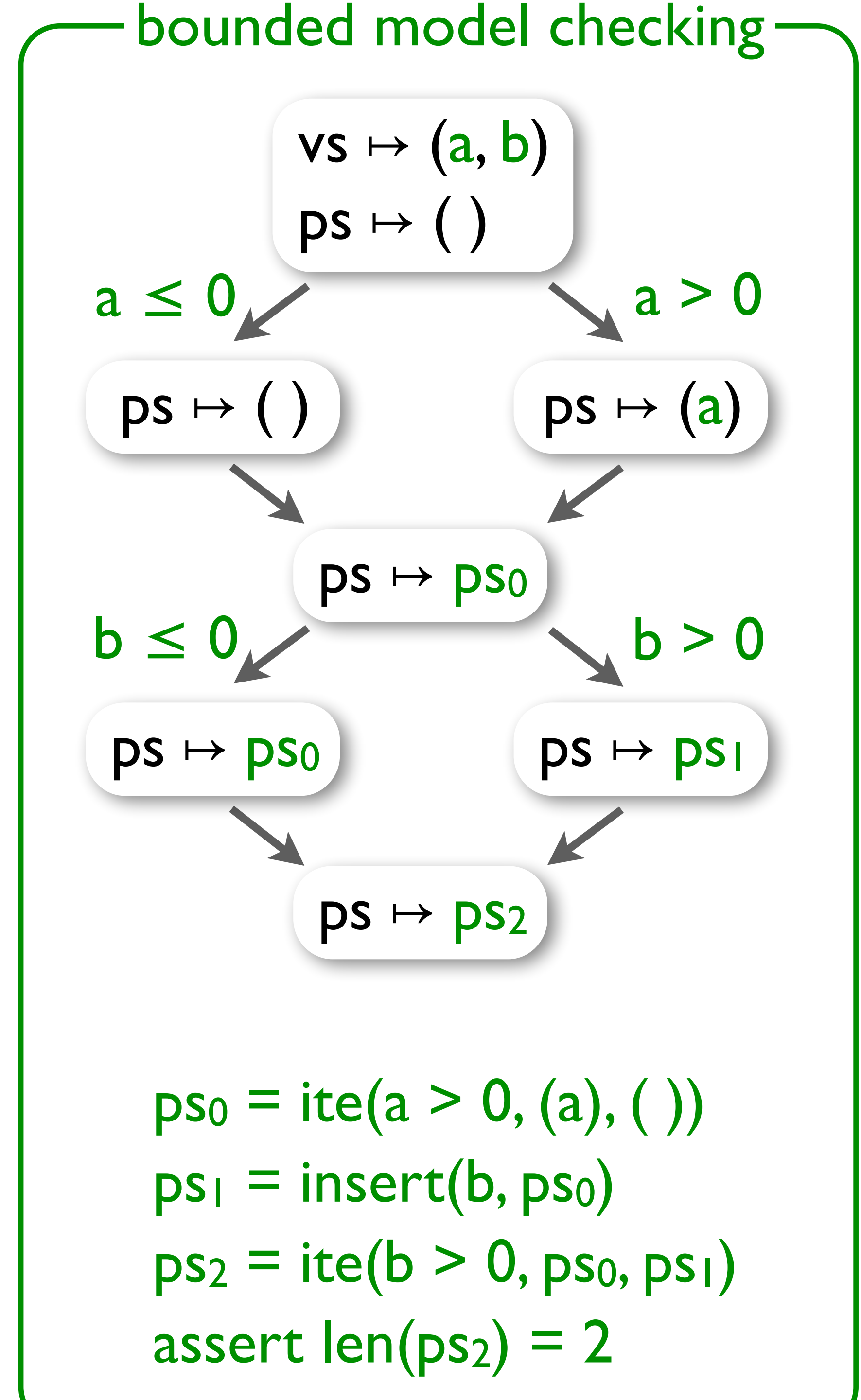
symbolic execution

vs ↦ (a, b)
ps ↦ ( )

$a \leq 0$ → ps ↦ ( )
$a > 0$ → ps ↦ (a)

$b \leq 0$ → ps ↦ ( )
$b > 0$ → ps ↦ (b)
$b \leq 0$ → ps ↦ (a)
$b > 0$ → ps ↦ (b, a)

$$\begin{Bmatrix} a \leq 0 \\ b \leq 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a \leq 0 \\ b > 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b \leq 0 \\ \text{false} \end{Bmatrix} \vee \begin{Bmatrix} a > 0 \\ b > 0 \\ \text{true} \end{Bmatrix}$$



A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

a > 0
b > 0
true

A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

Merge values of

- primitive types: symbolically
- immutable types: structurally
- all other types: via unions

$\left\{ \begin{array}{l} a > 0 \\ b > 0 \\ \text{true} \end{array} \right\}$

A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

Merge values of

- primitive types: symbolically
- immutable types: structurally
- all other types: via unions

¬g
g
a
b
c

$\left\{ \begin{array}{l} a > 0 \\ b > 0 \\ true \end{array} \right\}$

A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

Merge values of

- primitive types: symbolically
- immutable types: structurally
- all other types: via unions

¬g
g
(a, b)
(c, d)
(e, f)

$\left\{ \begin{array}{l} a > 0 \\ b > 0 \\ true \end{array} \right\}$

A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

Merge values of

- primitive types: symbolically
- immutable types: structurally
- all other types: via unions

$\neg g$ g

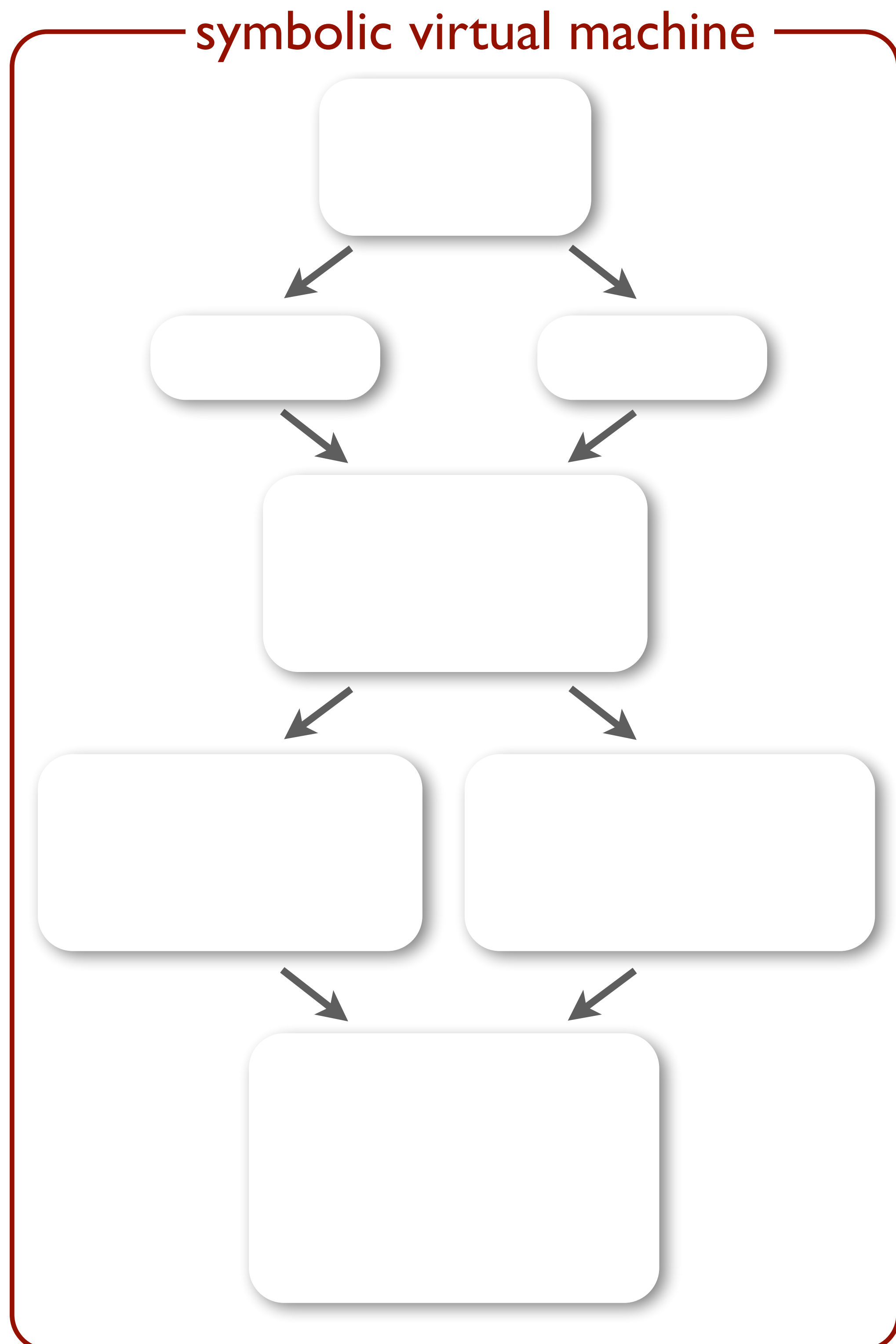
a   ()

$\{ \neg g \vdash a, g \vdash () \}$

$\left\{ \begin{array}{l} a > 0 \\ b > 0 \\ true \end{array} \right\}$

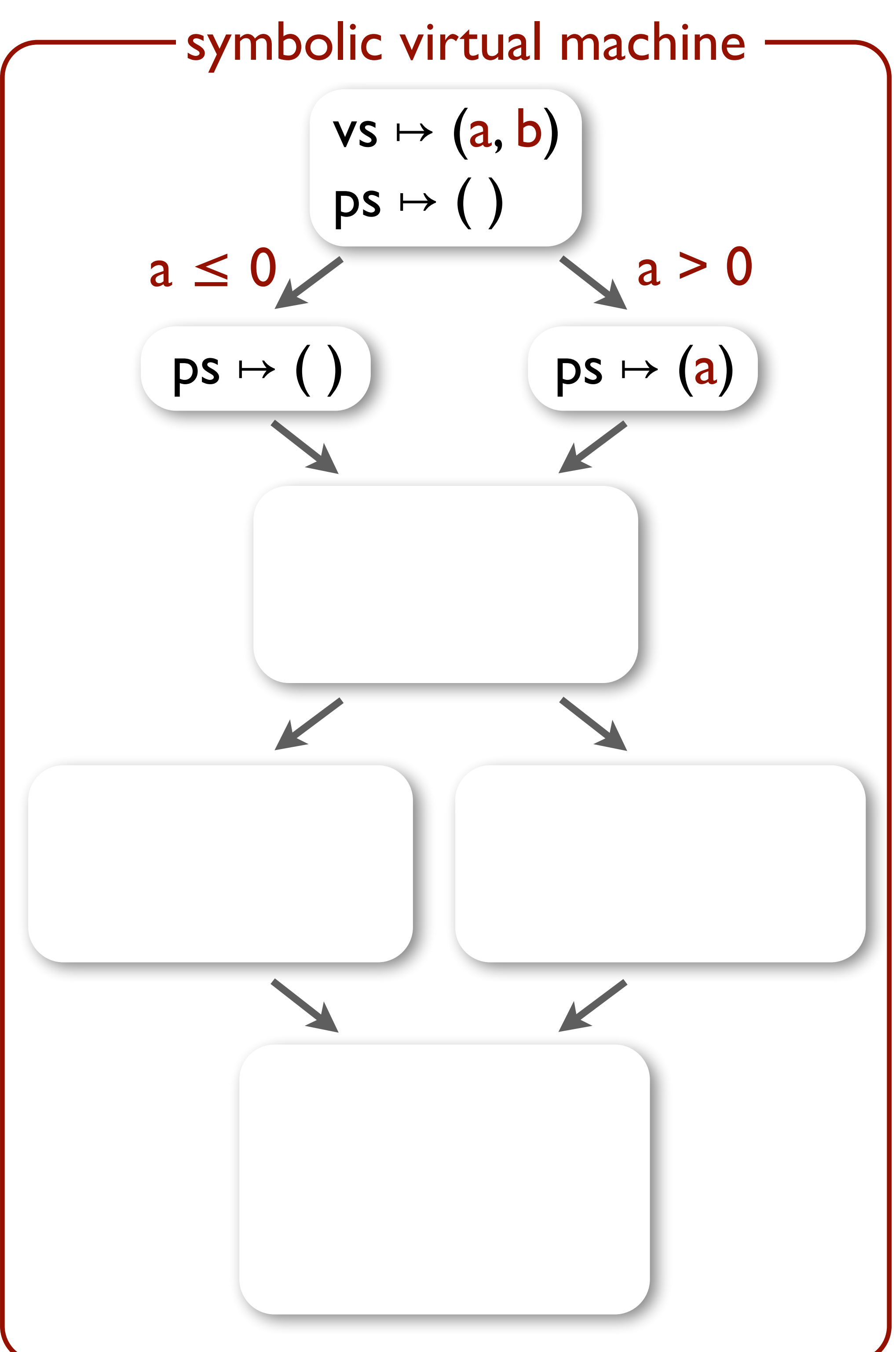
A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```



A new design: type-driven state merging

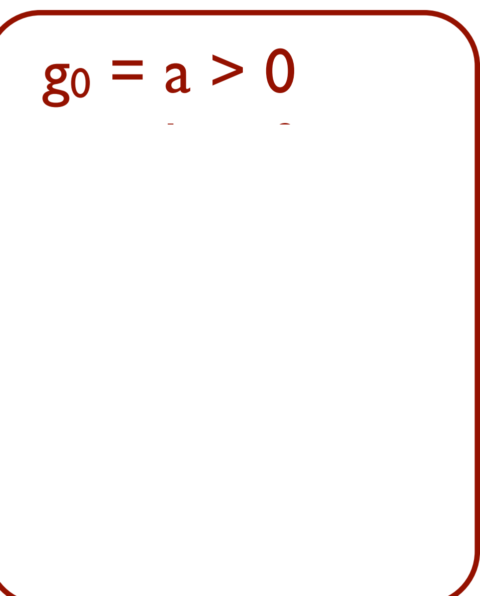
```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

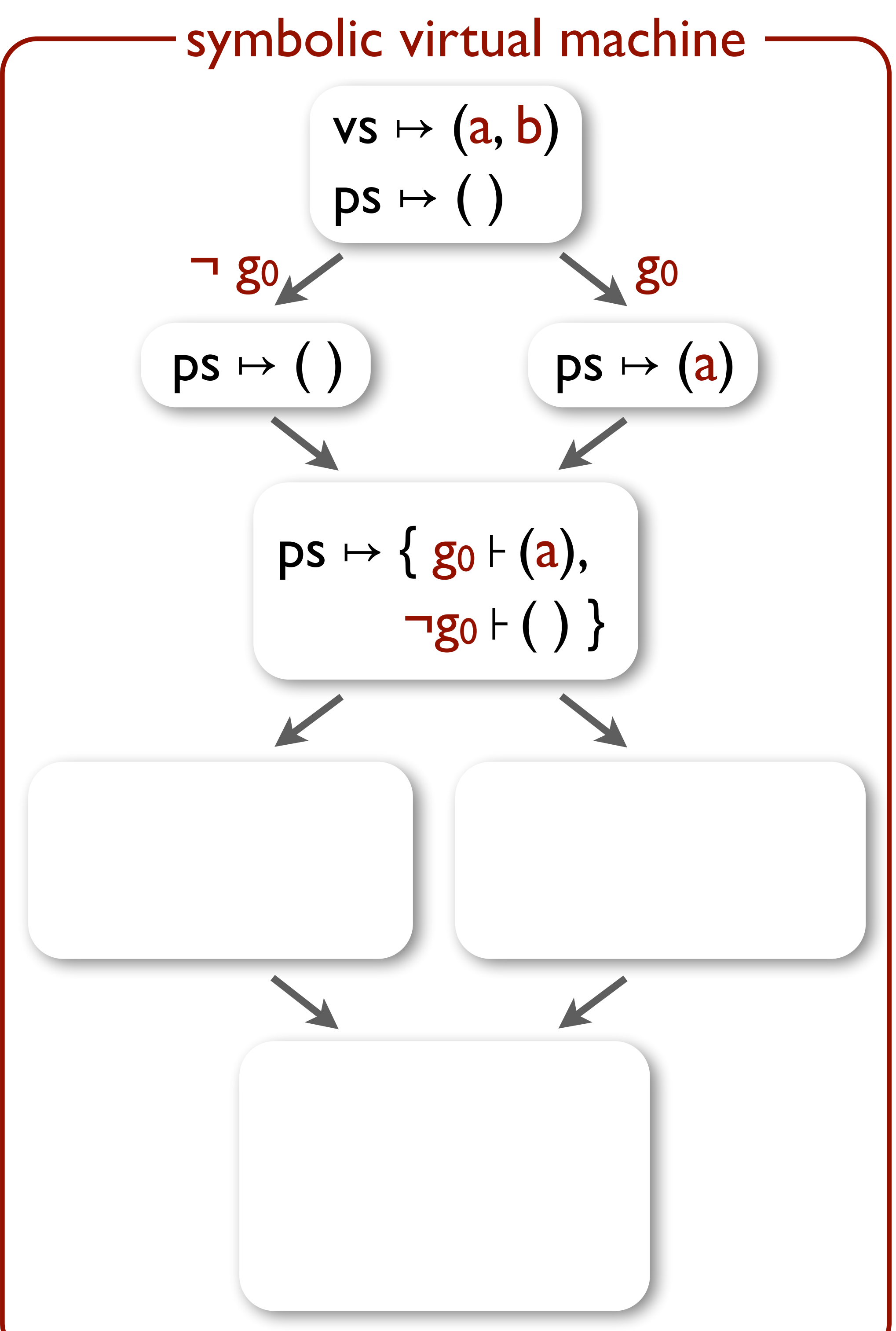


A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

Symbolic union: a set of guarded values, with disjoint and exhaustive guards.



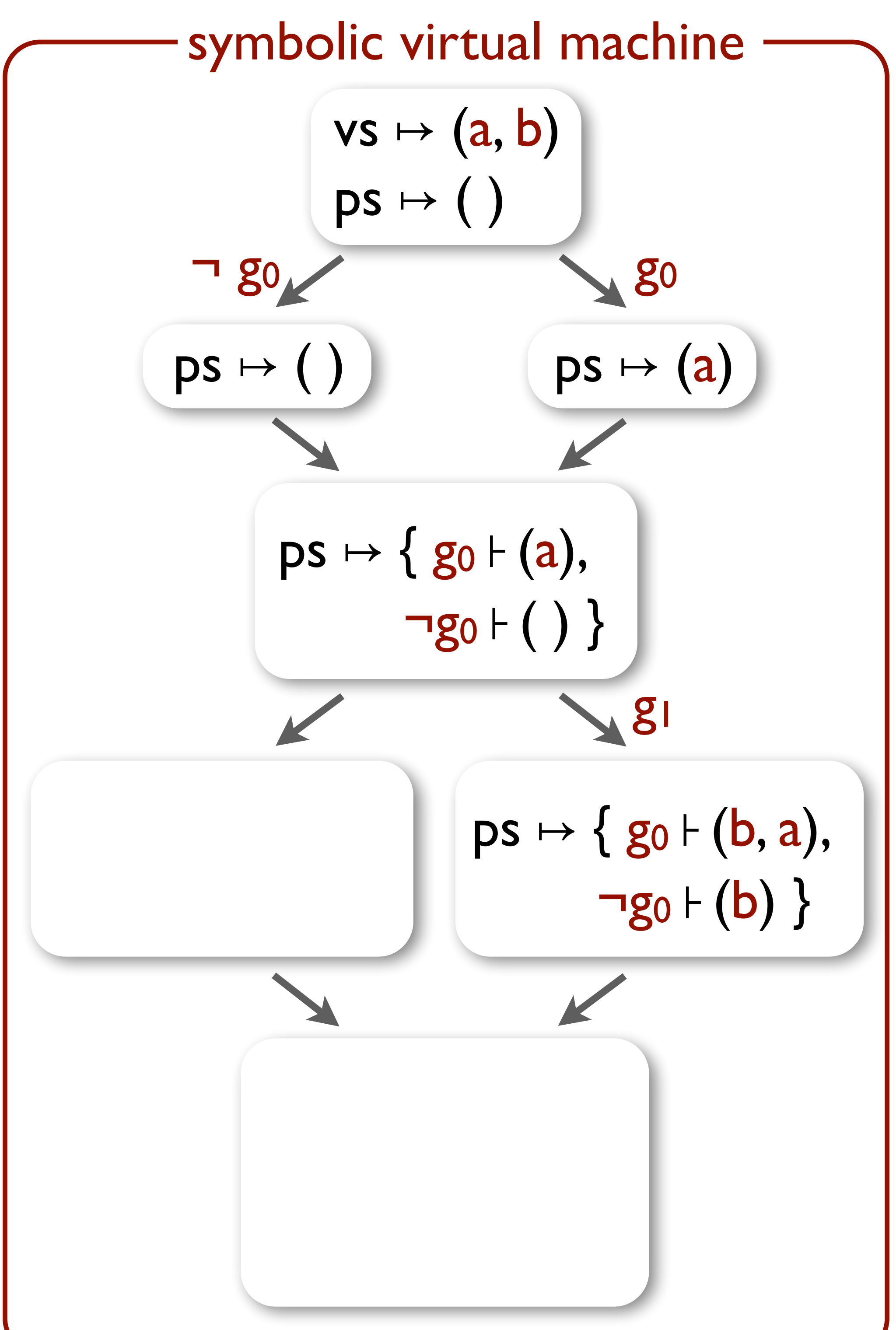


A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

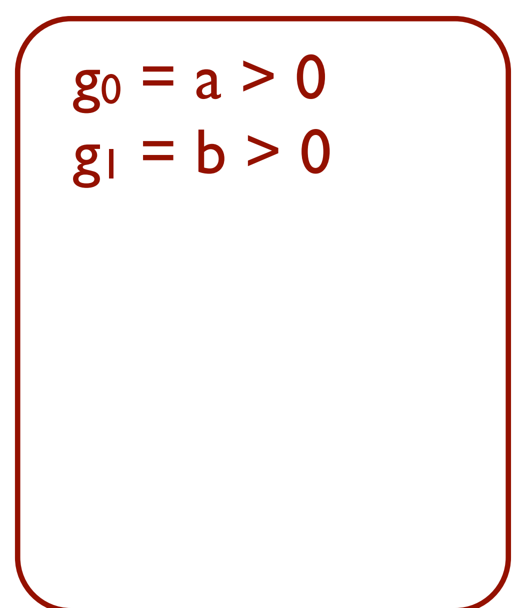
Execute `insert` concretely on all lists in the union.

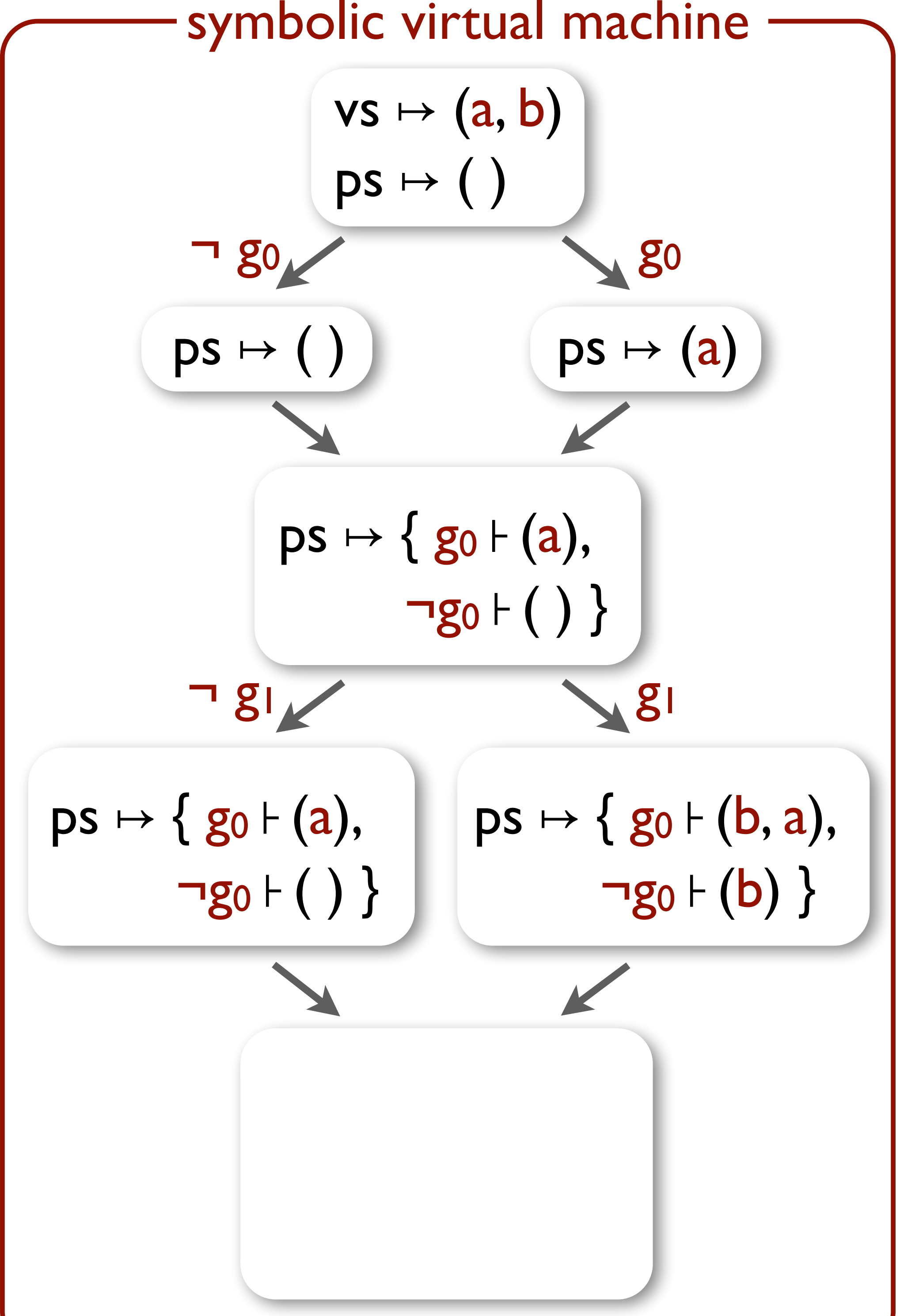
$g_0 = a > 0$
$g_1 = b > 0$



A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

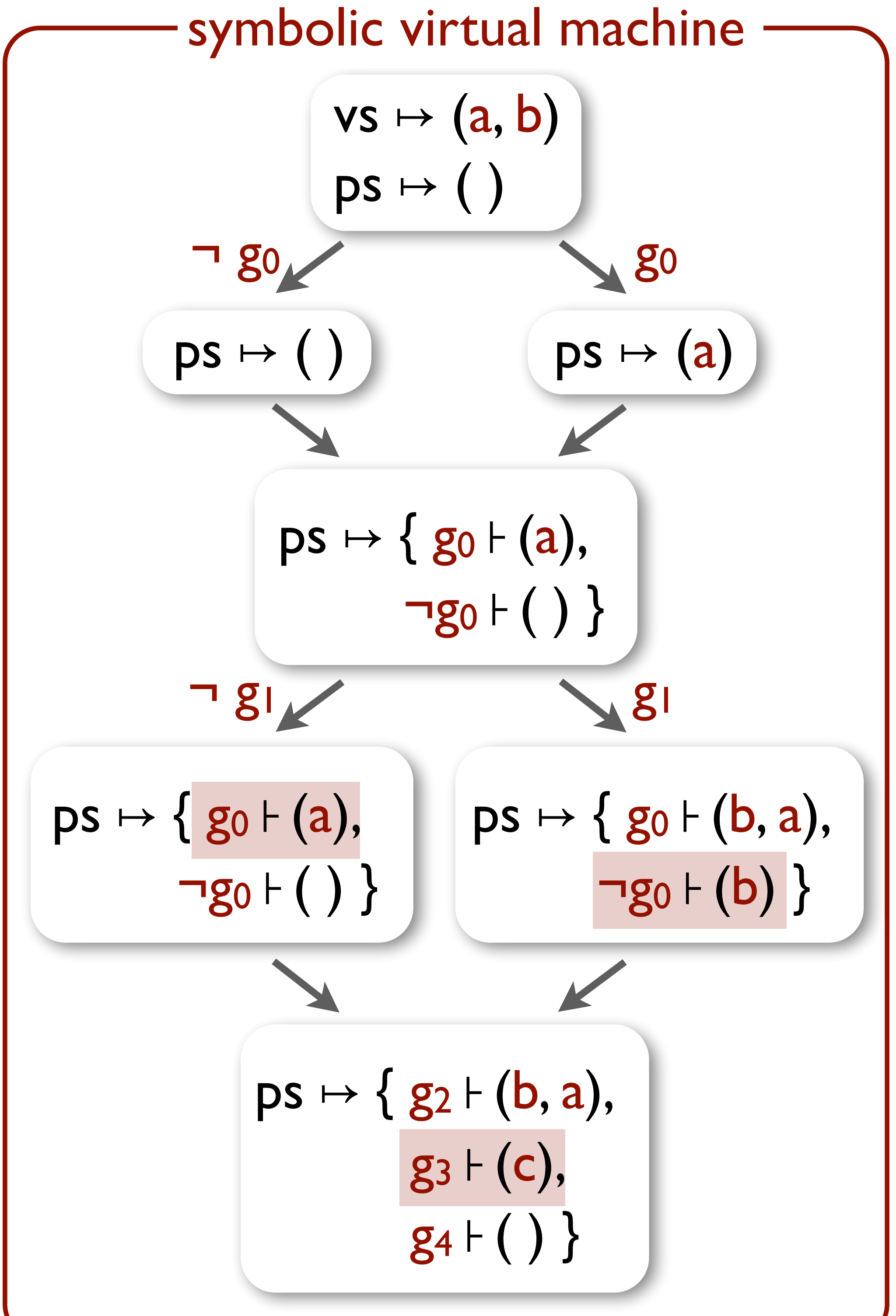




A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

$g_0 = a > 0$
$g_1 = b > 0$
$g_2 = g_0 \wedge g_1$
$g_3 = \neg(g_0 \Leftrightarrow g_1)$
$g_4 = \neg g_0 \wedge \neg g_1$
$c = \text{ite}(g_1, b, a)$

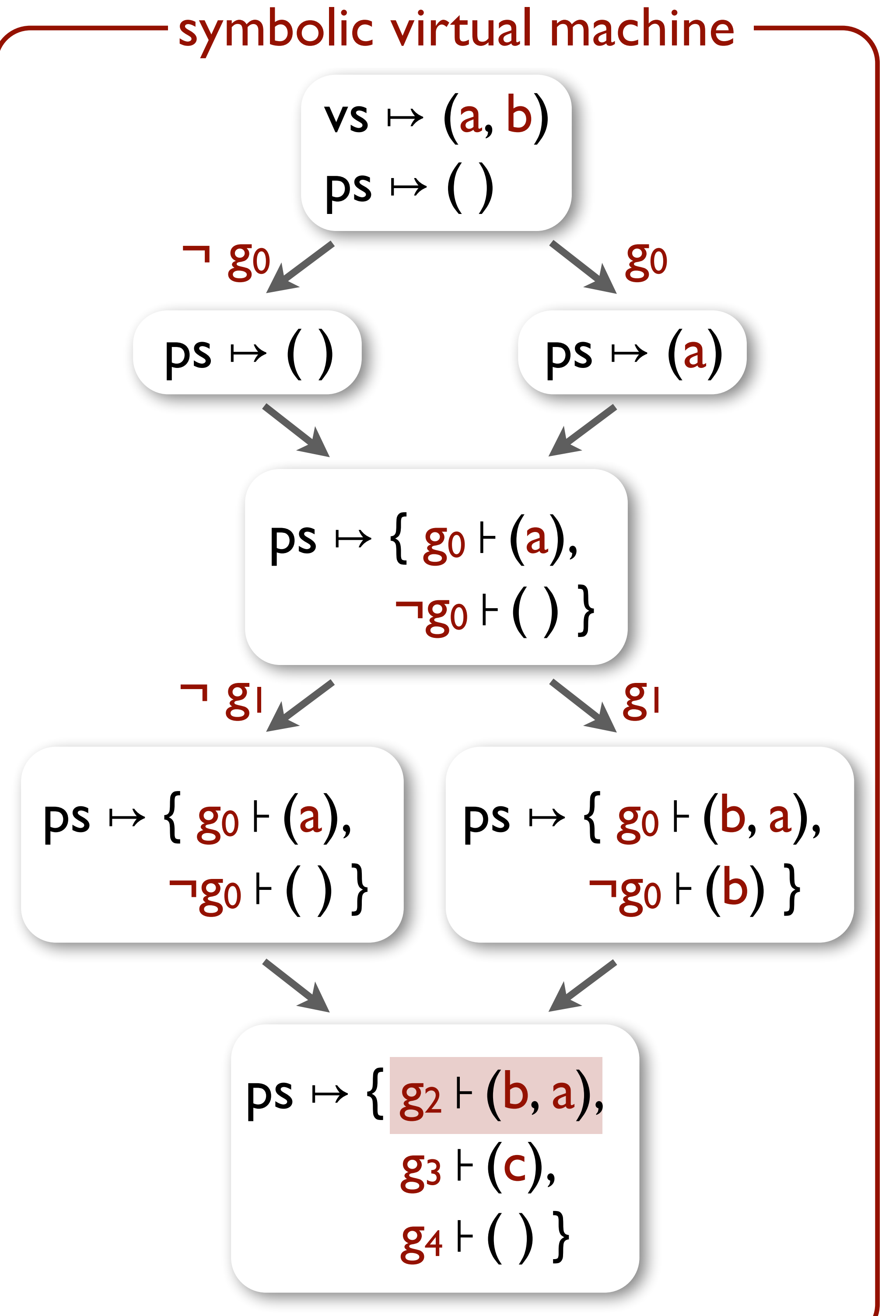


A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

Evaluate `len` concretely on all lists in the union; assertion true only on the list guarded by g_2.

$g_0 = a > 0$
$g_1 = b > 0$
$g_2 = g_0 \wedge g_1$
$g_3 = \neg(g_0 \Leftrightarrow g_1)$
$g_4 = \neg g_0 \wedge \neg g_1$
$c = \text{ite}(g_1, b, a)$
assert g_2



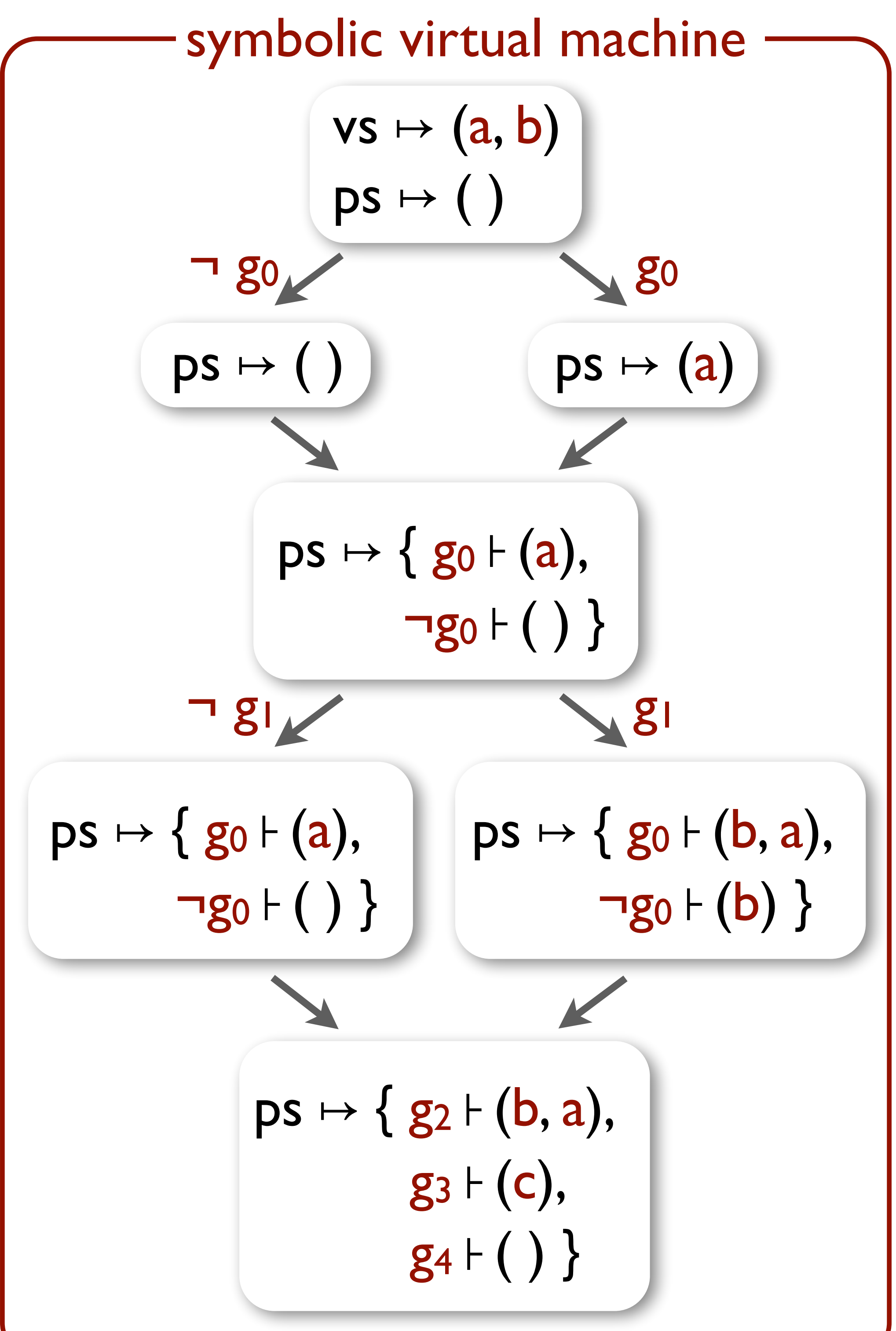
A new design: type-driven state merging

```
solve:
  ps = ()
  for v in vs:
    if v > 0:
      ps = insert(v, ps)
  assert len(ps) == len(vs)
```

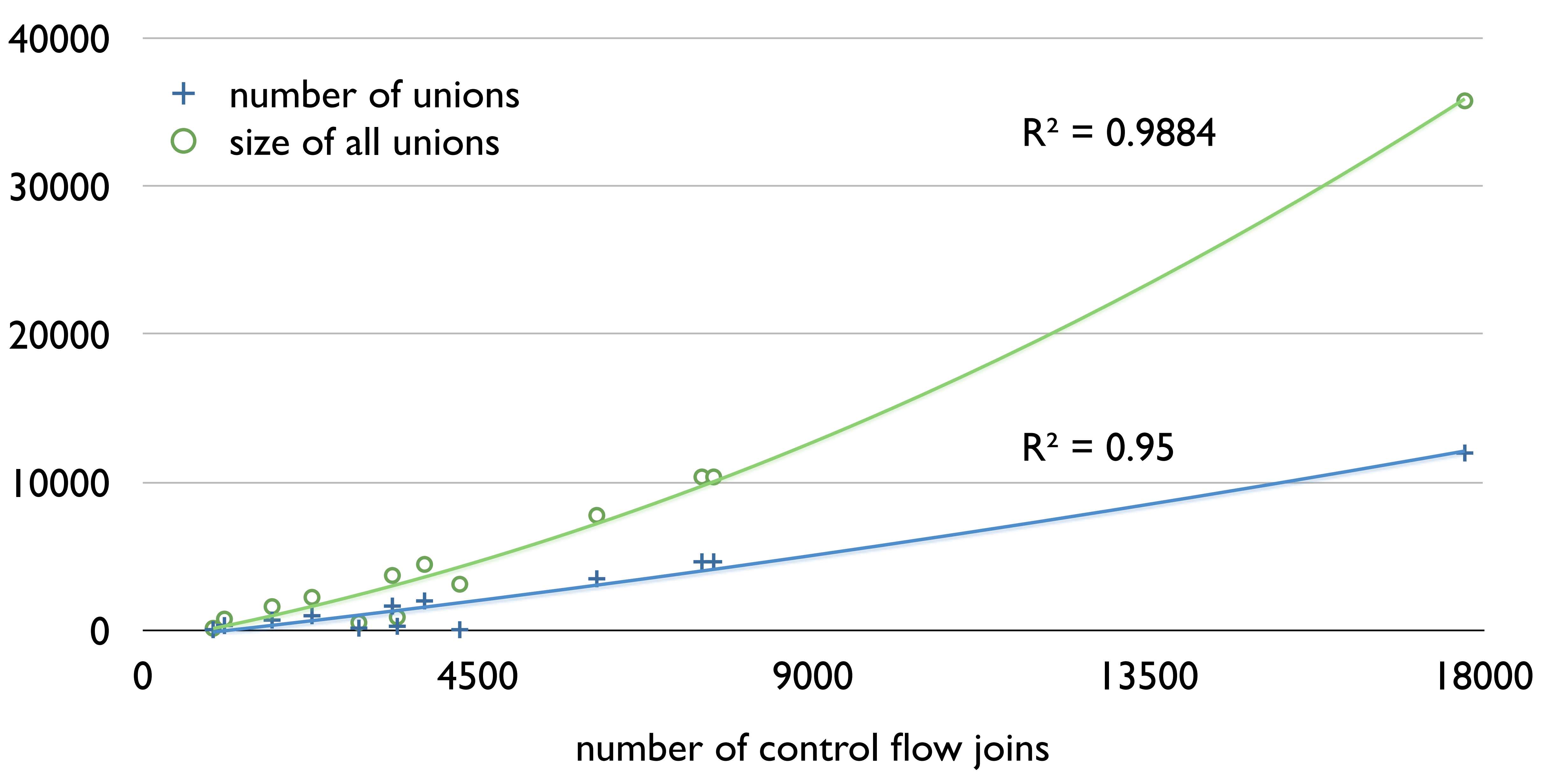
polynomial encoding

partial evaluation

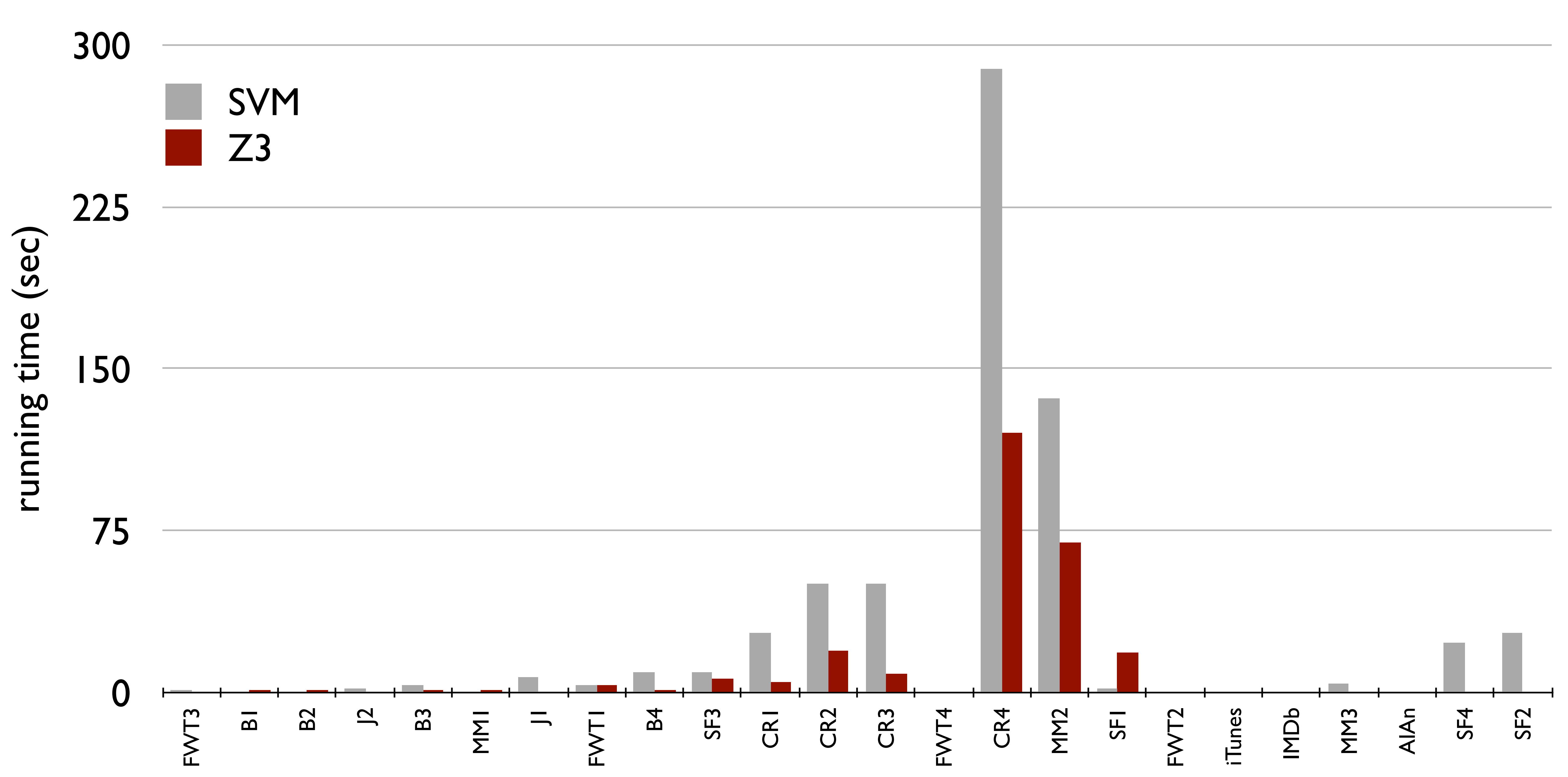
$g_0 = a > 0$
$g_1 = b > 0$
$g_2 = g_0 \wedge g_1$
$g_3 = \neg(g_0 \Leftrightarrow g_1)$
$g_4 = \neg g_0 \wedge \neg g_1$
$c = \text{ite}(g_1, b, a)$
assert g_2



Effectiveness of type-driven state merging

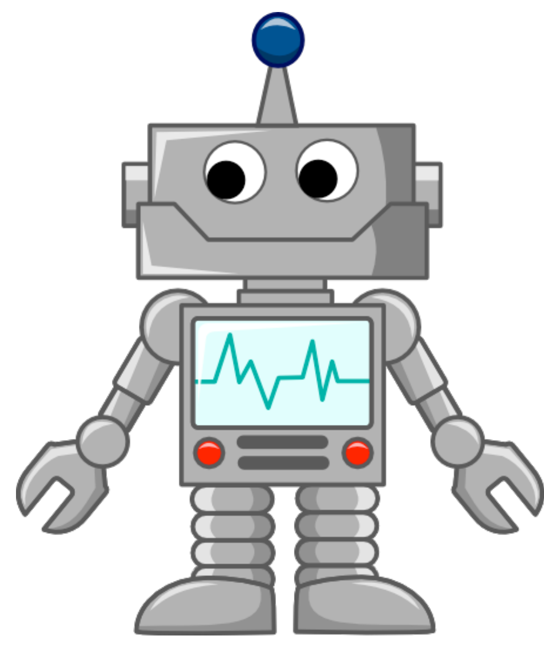


Effectiveness of type-driven state merging



demo

a little SDSL for finite state automata



thanks

emina@eecs.berkeley.edu