

Mechanical Verification of Avicenna's Proof on the Existence of a Necessary Existent and Its Unity

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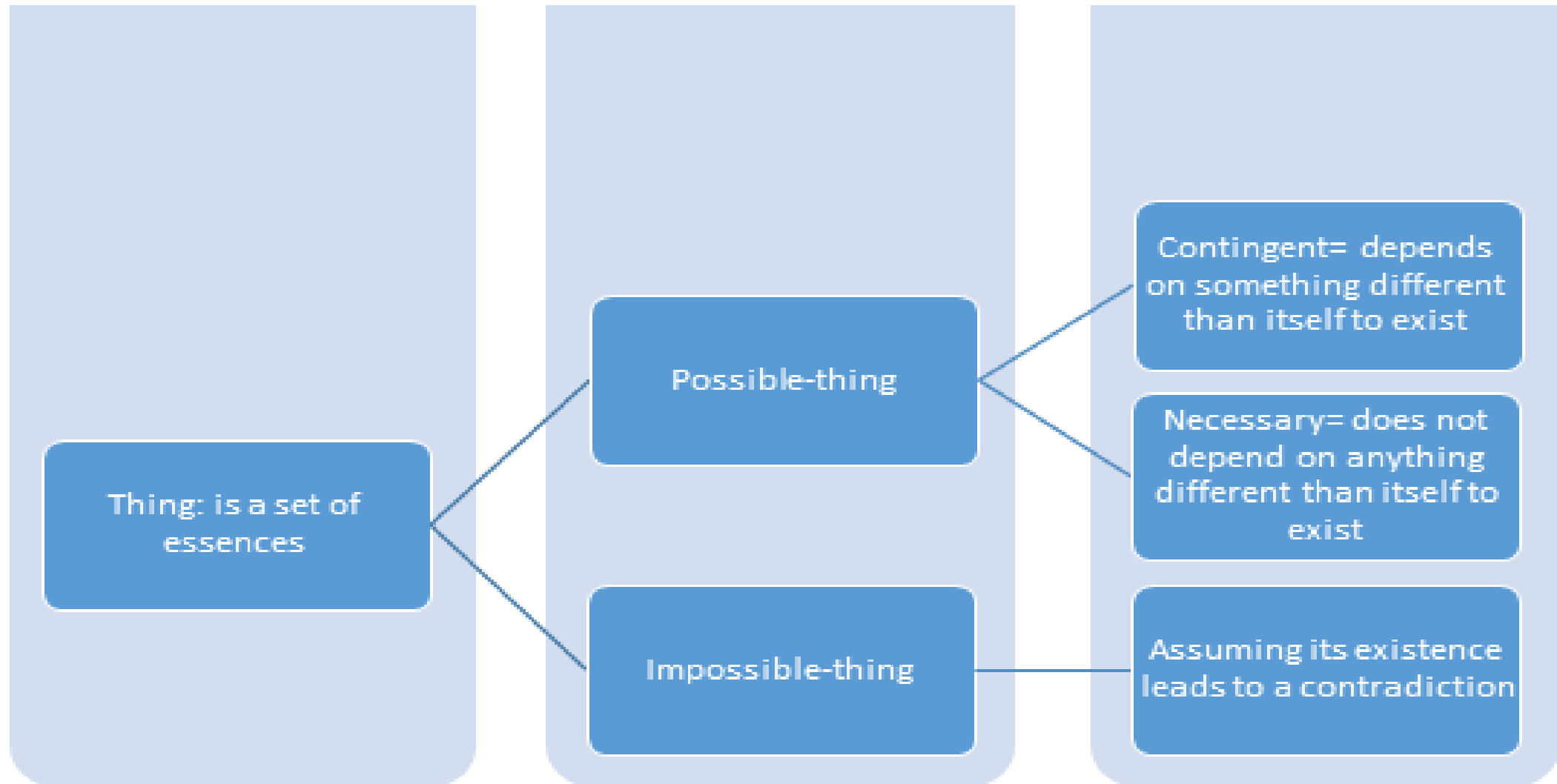
Quotation:

- *“ I started my book “Al Najat” by an introduction to Logic since it is the shell that can protect our minds from going astray. Also it is the mechanical tool we use to reach the truth by following its reasons and proofs”.*

Avicenna, Al Najat- In Logical Wisdom and God’s Nature.

في كتابي "النجات" بدأت بكتاب في المنطق لأنه القشرة التي تحمي عقولنا من الضلال. كما أنه الأداة الميكانيكية التي نستخدمها للوصول إلى الحقيقة باتباع أسبابها وبراهينها.

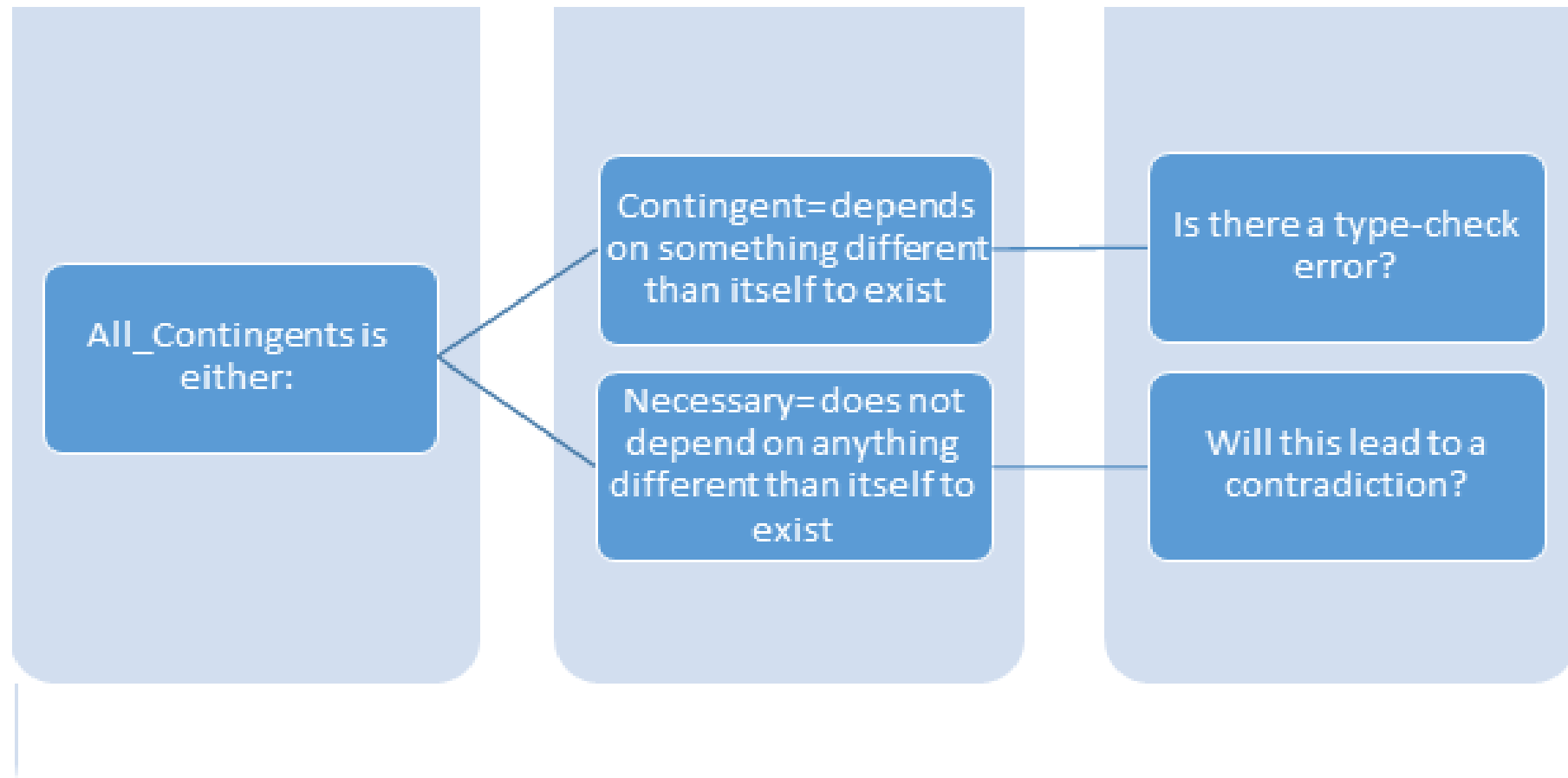
Avicenna's Hierarchy of Things:



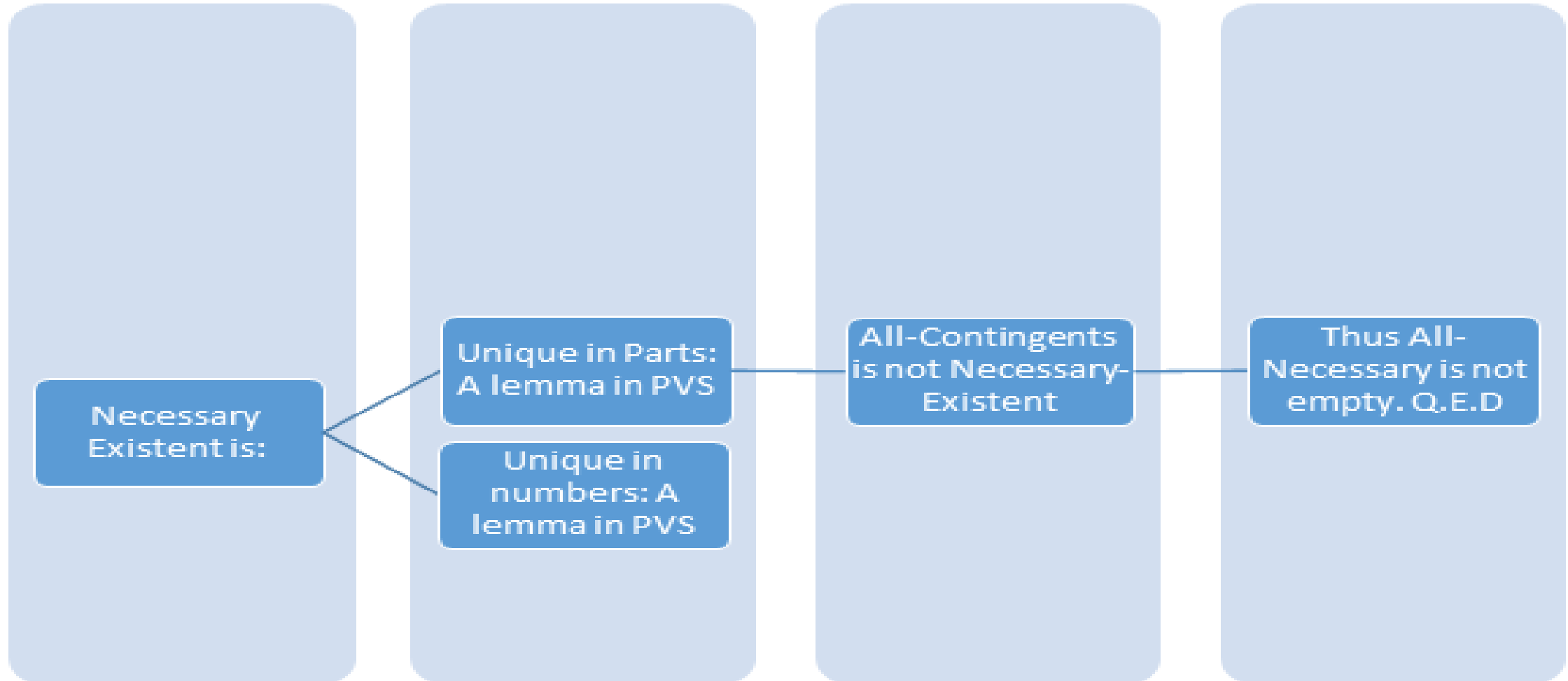
Avicenna's Main Claim:

Set of All-Contingents is contingent;
hence the existence of a Necessary
Existent.

Criticisms:



Avicenna's Proof Sketch:



PVS “Unique In Parts” Theorem

```
Mandatory: essence
s1: VAR essence
x1: VAR Necessary Existent

Necessary is Unique in parts: THEOREM

    not member(Mandatory ,singleton(s1)) and member(s1,x1) IMPLIES

    Exists (x3:possible_thing): member(x3, Cause(x1)) and diff?(x1, x3)
```

If x1 has at least two different parts: in the simplest form {s1} and Mandatory then there will be a possible thing different than x1 that is causing x1 which means x1 is contingent! A contradiction.

PVS “Unique in Numbers” Theorem

```
x1, x2: VAR Necessary Existent
```

```
Necessary_is_unique_innumber_3: THEOREM diff?(x1,x2) IMPLIES
```

```
Exists (x3:possible_thing): member(x3,Cause(x1)) and diff?(x1, x3)
```

This means x1 has depends on x3 which is different than itself. A contradiction!

Major Criticism assumption generates Unprovable TCC

```
% NecessaryExists_0_TCC1 :  
  
% |-----  
% {1}  FORALL (f: set[Necessary]):  
  
% f = All_Necessary IMPLIES (FORALL (x: set[Contingent]): singleton?[Contingent](x))
```

This TCC is unprovable and it was generated under the criticism assumption :
"There is no difference between a set and a type".

PVS Model and Proof:

- Note : Complete .pvs and .prf are available on our website (please copy paste the link it if it did not work):

<http://asd.cs.mtu.edu/projects/mechVerif/specs.html>

Contributions:

- Mechanical proof of Avicenna's proof using PVS
- Identifying the type check error (mentioned in criticisms) using PVS
- Mechanical proof of the existence of a Necessary Existent and the proof of its Unity
- A thousand years ago, Avicenna knew the difference between a TYPE "نوع" and a set which may contain elements of different types "جملة او "جميع القائم بالغير"

Acknowledgment

- Organizers of the 4th summer school in formal techniques for giving us this opportunity to have this discussion.