1. Given the formula
$$(a_1 \wedge b_1) \vee (a_2 \wedge b_2) \vee \ldots \vee (a_{20} \wedge b_{20})$$
Is it feasible to convert it into an equivalent CNF formula? Convert it into an equisatisfiable CNF formula using auxiliary variables. Prove they are indeed equisatisfiable using Yices or Z3.

2. Many problems can be easily encoded using the constraint *at-most-one*$(a_1, \ldots, a_n)$. This constraint is satisfied if at most one of the Boolean variables is true. Encode the constraint
$$\textit{at-most-one}(a_1, a_2, a_3, a_4, a_5)$$
into an equivalent set of clauses. Try it using Yices or Z3. How many clauses do you need to encode an *at-most-one* constraint containing n arguments?

3. The pigeon-hole problem is a classic satisfiability problem. We use the Boolean variable $p_{i,j}$ to represent that pigeon i is at hole j. Assume we have $n+1$ pigeons and n holes, and we want to encode the following constraints:
   - Each pigeon is in some hole.
   - Two different pigeon cannot be in the same whole.

   Show this problem is unsatisfiable using Yices or Z3 (use $n = 8$ and $n = 16$). The idea is to write a script language (e.g., Python) to generate the formulas. Encode this problem in arithmetic using 0-1 variables, and a conjunction of arithmetic equalities/inequalities (avoid disjunctions). Try the new encoding using Yices or Z3.

4. Show that any CNF formula can be encoded into an equisatisfiable formula using a conjunction of literals in the theory of integer difference logic and uninterpreted function symbols. Encode the formula
$$(a \vee b) \wedge (a \vee \neg b) \wedge (\neg a \vee b) \wedge (\neg a \vee \neg b)$$
and try it using Yices or Z3.

5. The if-then-else term $ite(c,t,e)$ is used to encode several problems in practice. If $c = true$, then $ite(c,t,e) = t$. Otherwise, $ite(c,t,e) = e$. Given the formula
$$\begin{array}{l} x_1 = ite(c_1, y+1, y-1) \wedge x_2 = ite(c_2, y+1, y-1) \wedge \ldots \wedge x_{16} = ite(c_{16}, y+1, y-1) \wedge \\ s = x_1 + x_2 + \ldots + x_{16} \wedge \neg(y - 16 \leq s \wedge s \leq y + 16) \end{array}$$
Show that it is unsatifiable using Yices or Z3. Create an equisatisfiable formula that does not use *ite*.

6. Given the formula
$$\begin{array}{l} ((x_1 = 0 \wedge y_1 = 0) \vee (x_1 = 1 \wedge y_1 = 1)) \wedge \\ ((x_2 = 0 \wedge y_2 = 0) \vee (x_2 = 1 \wedge y_2 = 1)) \wedge \\ \ldots \\ ((x_{16} = 0 \wedge y_{16} = 0) \vee (x_{16} = 1 \wedge y_{16} = 1)) \wedge \\ f(x_1, f(x_2, \ldots f(x_{15}, x_{16}))) \neq f(y_1, f(y_2, \ldots f(y_{15}, y_{16}))) \end{array}$$
Show that it is unsatisfiable using Yices or Z3. Create an equisatisfiable formula that does not contain the unintepreted function symbol f.