Some basic elements of logic

Safety argumentation seen from a logical perspective

Toulmin’s argument patterns

GSN-based approaches to safety argumentation
entailment relation \( \Gamma \vdash \varphi \)
Some Elements of Logic

object language \( \text{Sen} \)

entailment relation \( \Gamma \vdash \varphi \)
Some Elements of Logic

object language: Sen
entailment relation: $\Gamma \vdash \varphi$
inference system: InfSys
Some Elements of Logic

object language $\text{Sen}$
entailment relation $\Gamma \vdash \varphi$
inference system $\text{InfSys}$
proof

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Some Elements of Logic

object language  \textit{Sen}

entailment relation  \( \Gamma \vdash \varphi \)

inference system  \textit{InfSys}

proof

propositional language  \textit{Prop}
Some Elements of Logic

<table>
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<td>proof</td>
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propositional language | Prop |

classical propositional logic | $\{ \neg p \lor \neg q \} \vdash_{CPL} \neg(p \land q)$ |
**Some Elements of Logic**

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<td>![Proof Diagram]</td>
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**Some Elements of Logic**

- **object language** \( \text{Sen} \)
- **entailment relation** \( \Gamma \vdash \varphi \)
- **inference system** \( \text{InfSys} \)
- **proof**

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- **propositional language** \( \text{Prop} \)
- **classical propositional logic** \( \{ \neg p \lor \neg q \} \vdash_{\text{CPL}} \neg (p \land q) \)
- **inference system** \( \text{NatDed} \)
- **proof**

\[
\begin{array}{ccc}
   & (2) & (2) \\
\hline
   & (1) & (1) \\
\hline
p \land q & \neg p & \neg q \\
\hline
p & \neg p & q \\
\hline
\neg p \lor \neg q & \bot & \bot \\
\hline
\bot & \bot & \bot \\
\hline
\neg (p \land q) & (2) & (1)
\end{array}
\]
Some Elements of Logic

object language: \text{Sen} \\
entailment relation: \Gamma \vdash \varphi \\
inference system: \text{InfSys} \\
proof

propositional language: \text{Prop} \\
classical propositional logic: \{ \neg p \lor \neg q \} \vdash_{\text{CPL}} \neg (p \land q) \\
inference system: \text{NatDed} \\
proof

\[ \frac{p \land q}{p} \quad \frac{p}{\neg p} \quad \frac{p \land q}{q} \quad \frac{q}{\neg q} \]

\[ \frac{\neg p \lor \neg q}{\perp} \quad \frac{\perp}{\neg (p \land q)} \]

Analyzeability
Some Elements of Safety Argumentation
Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \models_{S} \chi \)
Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \vdash_s \chi \)

inference system \( \text{InfSys}_s \)
Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \models_S \chi \)

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Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \vdash_s \chi \)

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proof

Is it possible to complete the picture classically?
Some Elements of Safety Argumentation

entailment relation $\mathcal{P} \vdash_S \chi$

inference system InfSys$_S$

proof

Is it possible to complete the picture classically?
Some Elements of Safety Argumentation

entailment relation  \( \mathcal{P} \vdash_S \chi \)

inference system  \( \text{InfSys}_S \)

\( \mathcal{P} \)  \( \vdash \)  \( \mathcal{P} \)  premisses are to be grounded in evidence

Is it possible to complete the picture classically?
Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \vdash_S \chi \)

inference system InfSys\(_S\)

proof

Is it possible to complete the picture classically?

\( \mathcal{P} \) premisses are to be grounded in evidence

InfSys\(_S\) what are sensible rules of inference?
Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \vdash_S \chi \)

inference system \( \text{InfSys}_S \)

proof

premises are to be grounded in evidence

what are sensible rules of inference?

what is meant by proof?

Is it possible to complete the picture classically?
Some Elements of Safety Argumentation

- entailment relation: $\mathcal{P} \vdash_S \chi$
- inference system: $\text{InfSys}_S$

$\mathcal{P}$
- premisses are to be grounded in evidence
- $\text{InfSys}_S$
- what are sensible rules of inference?

Proof:
- what is meant by proof?

Answering these questions involves dealing with:

Is it possible to complete the picture classically?
Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \vdash_{S} \chi \)

inference system \( \text{InfSys}_{S} \)

proof

\( \mathcal{P} \) premisses are to be grounded in evidence

\( \text{InfSys}_{S} \) what are sensible rules of inference?

what is meant by proof?

Answering these questions involves dealing with:

Fallacies

Defeasibility

Is it possible to complete the picture classically?
Some Elements of Safety Argumentation

entailment relation \( \mathcal{P} \vdash S \chi \)

inference system \( \text{InfSys}_S \)

proof

\( \vdash \)

Premises are to be grounded in evidence

InfSys\( _S \)

What are sensible rules of inference?

What is meant by proof?

Answering these questions involves dealing with:

- Fallacies
- Judgement from Expert Opinion
- Defeasibility

Is it possible to complete the picture classically?
Some Elements of Safety Argumentation

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what are sensible rules of inference?

\( \mathcal{P} \)
what is meant by proof?

Answering these questions involves dealing with:

- Fallacies
- Judgement from Expert Opinion
- Defeasibility

\( \mathcal{P} \vdash_S \chi \quad \mathcal{P} \leq \mathcal{P}' \quad \mathcal{P}' \not\vdash_S \chi \)

Is it possible to complete the picture classically?
Toulmin’s Argument Patterns
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What is Toulmin’s notion of an argument pattern?
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Conceptual tool for analyzing rules of inference.
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Conceptual tool for analyzing rules of inference.

Toulmin’s Argument Patterns

- Premisses
- Data
- Rule of inference
- Warrant
- Rebuttal
- Exception to the warrant
- Qualifier
- Modal logical connective
- Claim
- Consequent

[Diagram of Toulmin's Argument Patterns]
What is Toulmin’s notion of an argument pattern?

Conceptual tool for analyzing rules of inference.
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Meta-(Meta-Notion)
What is Toulmin’s notion of an argument pattern?

**Conceptual tool for analyzing rules of inference.**

What is the logical status of Toulmin’s notion of an argument pattern?

**Meta-(Meta-Notion)**

What is Toulmin’s notion of an argument pattern good for?
Toulmin’s Argument Patterns

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It provides a basis for formulating rules of inferences for safety argumentation.
What is Toulmin’s notion of an *argument pattern*?

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GSN Diagrams

Back ing

Data

Warrant

Qualifier

Claim

Rebuttal
GSN Diagrams

Goal

Justification

Strategy

Sub-Goal

Qualifier

Rebuttal
GSN Diagrams

Goal

Justification

Strategy

Q

Sub-Goal

Rebuttal
Rebuttal

?
GSN Diagrams

- Goal
- Strategy
  - Justification
  - Rebuttal
  - Sub-Goal
- Solution
GSN Diagrams

Goal

Rebuttal

Justification

Strategy

Sub-Goal

Solution
What is captured by GSN diagrams?

GSN Diagrams

Goal

Justification

Strategy

Rebuttal

Sub-Goal

Evidence
What is captured by GSN diagrams?

Safety goal structured design
What is captured by **GSN diagrams**?

**Safety goal structured design**

Are GSN diagrams really capturing **safety arguments**?
What is captured by GSN diagrams?

Safety goal structured design

Are GSN diagrams really capturing safety arguments?
GSN Diagrams

What is captured by GSN diagrams?

Safety goal structured design

Are GSN diagrams really capturing safety arguments?

Are GSN diagrams logically useless?
What is captured by **GSN diagrams**?

Safety goal structured design

- **Goal**
- **Strategy**
- **Sub-Goal**
- **Evidence**
- **Justification**
- **Rebuttal**

Are GSN diagrams really capturing **safety arguments**?

Are GSN diagrams **logically useless**?
What is captured by GSN diagrams?

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proof strategies vs. proofs
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**proof strategies vs. proofs**
Summary

Presented some basic elements of logic as a science of reasoning.

Commented on what safety argumentation looks like when seen logically.

Discussed the logical status of Toulmin’s argument patterns.

Discussed the role of GSN diagrams in the context of safety argumentation.
“Can you suggest any fallacy?”

“He could not have fractured his skull in a fall?”

“In a morass, Watson?”

“I am at my wit's end”.

“Tut, tut; we have solved some worse problems.

At least we have plenty of material, if we can only use it”.

Sir Arthur Conan Doyle - The Return of Sherlock Holmes