PVS Exercises

Many of these exercises are in the NASA library, try to develop your own specifications and proofs. Google and Wikipedia are useful as well, if you haven't seen these proofs before.

1. Prove your favorite sorting algorithm is correct. Note that you need to come up with an invariant, and make it a part of the recursive call that sorts an array or list. See `two_way_sort.pvs` for an example of this.

2. Specify and prove the binary search algorithm to find the position of a target value within a sorted array.

3. Specify and prove that the McCarthy 91 function terminates:

$$M(n) = \begin{cases} n - 10 & \text{if } n > 100 \\ M(M(n+1)) & \text{otherwise} \end{cases}$$

4. State and prove the pigeonhole principle.

5. Show that the square root of 2 is irrational. Generalize this to any non-square natural number.

6. Show that any natural number has a unique prime factorization.

7. Show that the set of primes is infinite.

8. State and prove Cantor's theorem: for any set A, the set of all subsets of A (the power set of A) has a strictly greater cardinality than A itself. Don't define cardinality, simply show that there is no bijective function between them.

9. Show that the rational numbers are denumerable, i.e., there is a bijective function between the natural numbers and the rationals.

10. Show that the reals are nondenumerable. There are several approaches to this. The easier is to assume every real between 0 and 1 can represented as a sequence of digits, (or for a binary representation, a set of natural numbers can be used). This uses Cantor's diagonalization argument.

11. Now prove that the representation chosen is a model for the reals as given axiomatically in PVS. This is difficult, and requires the use of theory interpretations and Cauchy sequences or Dedekind cuts.