Mechanical Verification of Avicenna’s Proof on the Existence of a Necessary Existent and Its Unity

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"I started my book “Al Najat” by an introduction to Logic since it is the shell that can protect our minds from going astray. Also it is the mechanical tool we use to reach the truth by following its reasons and proofs”.

Avicenna, Al Najat- In Logical Wisdom and God’s Nature.
Avicenna’s Hierarchy of Things:

- **Thing**: is a set of essences
- **Possible-thing**: depends on something different than itself to exist
- **Impossible-thing**: does not depend on anything different than itself to exist
- **Contingent**: assuming its existence leads to a contradiction
Avicenna’s Main Claim:

Set of All-Contingents is contingent; hence the existence of a Necessary Existent.
Criticisms:

All_Contingents is either:

- Contingent = depends on something different than itself to exist
- Necessary = does not depend on anything different than itself to exist

- Is there a type-check error?
- Will this lead to a contradiction?
Avicenna’s Proof Sketch:

Necessary Existent is:

Unique in Parts: A lemma in PVS

Unique in numbers: A lemma in PVS

All-Contingents is not Necessary-Existent

Thus All-Necessary is not empty. Q.E.D
PVS “Unique In Parts” Theorem

Mandatory: essence
s1: VAR essence
x1: VAR Necessary Existent

Necessary is Unique in parts: THEOREM

not member(Mandatory ,singleton(s1)) and member(s1,x1) IMPLIES

Exists (x3:possible_thing): member(x3, Cause(x1)) and diff?(x1, x3)

If x1 has at least two different parts: in the simplest form {s1} and Mandatory then there will be a possible thing different than x1 that is causing x1 which means x1 is contingent! A contradiction.
PVS “Unique in Numbers” Theorem

\[ x_1, x_2: \text{VAR Necessary Existent} \]

\[ \text{Necessary} \_\text{is}\_\text{unique}
\_\text{innumber} \_3: \text{THEOREM} \ \text{diff}\_?\(x_1, x_2) \ \text{IMPLIES} \]

\[ \exists (x_3: \text{possible\_thing}): \text{member}(x_3, \text{Cause}(x_1)) \ \text{and} \ \text{diff}\_?(x_1, x_3) \]

This means \( x_1 \) has depends on \( x_3 \) which is different than itself. A contradiction!
Major Criticism assumption generates Unprovable TCC

```plaintext
% NecessaryExists_0_TCC1 :
% |--------
% {1}    FORALL (f: set[Necessary]):
% f = All_Necessary IMPLIES (FORALL (x: set[Contingent]): singleton?[Contingent](x))
```

This TCC is unprovable and it was generated under the criticism assumption: "There is no difference between a set and a type".
PVS Model and Proof:

• Note: Complete .pvs and .prf are available on our website (please copy paste the link if it did not work):

   http://asd.cs.mtu.edu/projects/mechVerif/specs.html
Contributions:

• Mechanical proof of Avicenna’s proof using PVS
• Identifying the type check error (mentioned in criticisms) using PVS
• Mechanical proof of the existence of a Necessary Existent and the proof of its Unity
• A thousand years ago, Avicenna knew the difference between a TYPE "نوع" and a set which may contain elements of different types جمله او "جميع القائم بالغير"
Acknowledgment

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