Specification, Proof, and Model Checking of the Mondex Electronic Purse

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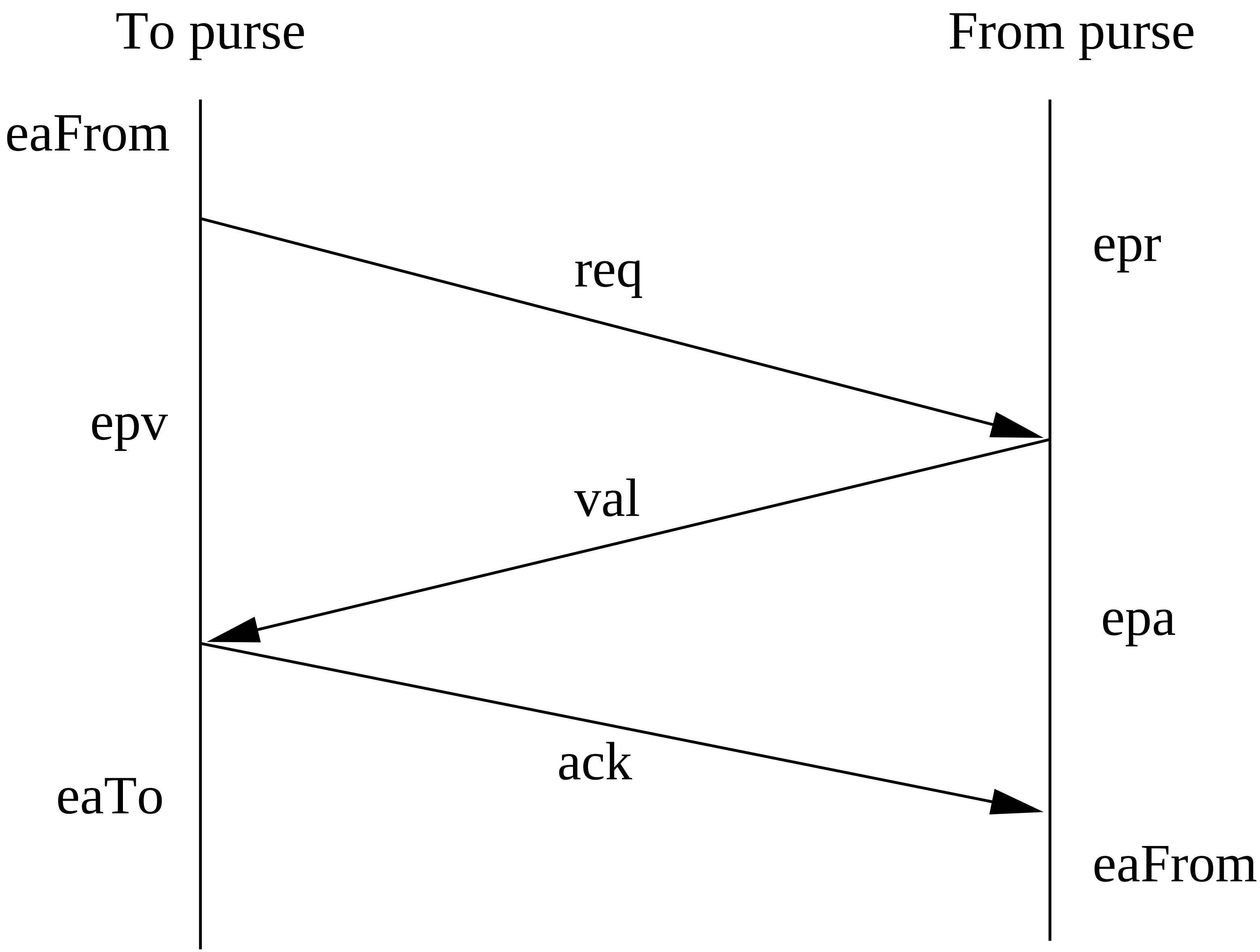

To purse
From purse
eaFrom
epr
req
epv
val
epa
ack
eaTo
eaFrom

The Problem

1. Specify the protocol in detail
2. Prove that each operation satisfies two conditions:
   (a) *NoValueCreation*:
   inPurses + *inTransit* never increases
   (b) *AllValueAccounted*:
   inPurses + *inTransit* + *lost* is constant

We will call such an operation *correct*.

The RAISE Approach

3 levels of specification:

1. Abstract: a problem in accounting. No purses; no messages; just three “bottom line” values and four abstract *correct* operations that transfer money between them.

2. Intermediate: abstract states and (complete set of) abstract operations. No details of the mechanisms that preserve the (asserted) invariant. Prove that each intermediate operation implements an abstract (and hence *correct*) operation.

3. Concrete: full details of the protocol. Prove that each operation implements its intermediate version.

Abstract Specification

4 abstract operations

TransferLeft

$$\begin{aligned}&inPurses' = inPurses - valu(m) \wedge \\ &(lost' = lost \wedge inTransit' = inTransit + valu(m) \vee \\ &\ lost' = lost + valu(m) \wedge inTransit' = inTransit)\end{aligned}$$

TransferRight

$$\begin{aligned}&inPurses' = inPurses + valu(m) \wedge \\ &lost' = lost \wedge \\ &inTransit' = inTransit - valu(m)\end{aligned}$$

Abort

$$\exists\, \mathsf{v} : \mathbf{Nat} \bullet$$
$$\mathsf{inPurses}' = \mathsf{inPurses} \wedge$$
$$\mathsf{lost}' = \mathsf{lost} + \mathsf{v} \wedge$$
$$\mathsf{inTransit}' = \mathsf{inTransit} - \mathsf{v}$$

No_op

$$\mathsf{inPurses}' = \mathsf{inPurses} \wedge$$
$$\mathsf{lost}' = \mathsf{lost} \wedge$$
$$\mathsf{inTransit}' = \mathsf{inTransit}$$

No_op is really a special case of Abort.

It is easy to prove these 4 operations are *correct*.

Key relations

$\text{totalCirculating} = \text{inPurses} + \text{inTransit}$

$\text{totalAccounted} = \text{inPurses} + \text{lost} + \text{inTransit}$

$\text{inPurses} = \Sigma_{\mathrm{balance}}(\text{purses})$

$\text{inTransit} = \Sigma_{\mathrm{valu}}(\text{toInEpv} \cap (\text{fromLogs} \cup \text{fromInEpa})$

$\text{lost} = \Sigma_{\mathrm{valu}}(\text{toLogs} \cap (\text{fromLogs} \cup \text{fromInEpa}))$

toInEpv/*fromInEpa* are the payment details of purses with status *epv*/*epa* respectively.

A payment details goes into *toLogs*/*fromLogs* if a purse aborts or restarts from status *epv*/*epa* respectively.

Intermediate Specification

- Two modules: PURSE1 and WORLD1
- Both abstract:
  - Abstract state types *Purse* and *World*
  - Function signatures
  - State invariants as axioms
  - Observer-generator axioms
- Collection of operations is complete
- States are incomplete (no purse logs, sequence numbers, archive)
- *fromLogs* and *toLogs* are abstract observers

Intermediate Specification: World: invariant

axiom

[isWorldAxiom]

$\forall$ w : World, p : P.Purse •

p $\in$ **rng** purses(w) $\Rightarrow$

(P.status(p) $=$ T.epr $\Rightarrow$

P.pdAuth(p) $\notin$ fromInEpa(w) $\wedge$

P.pdAuth(p) $\notin$ fromLogs(w) $\wedge$

(T.req(P.pdAuth(p)) $\in$ ether(w) $\Rightarrow$

P.pdAuth(p) $\in$ toInEpv(w) $\wedge$ P.pdAuth(p) $\notin$ toLogs(w) $\vee$

P.pdAuth(p) $\in$ toLogs(w) $\wedge$ P.pdAuth(p) $\notin$ toInEpv(w))) $\wedge$

... $\wedge$

visible(w) $\subseteq$ ether(w)

Concrete Specification

- Two modules: PURSE2 and WORLD2
- All types concrete and complete
- Almost all functions concrete; some intentional underspecification (loss of messages; increase in sequence numbers; purse payment details in *eaTo*, *eaFrom*) handled with axioms
- *fromLogs* and *toLogs* now a construction from purse logs and archive

Concrete Specification: World: invariant 1

$$
\begin{array}{l}
\text{isWorld} : \text{WorldBase} \rightarrow \mathbf{Bool} \\
\text{isWorld(w)} \equiv \\
\quad (\forall\ \text{n} : \text{Name} \bullet \\
\qquad \text{n} \in \mathbf{dom}\ \text{archive(w)} \Rightarrow \text{n} \in \mathbf{dom}\ \text{purses(w))} \wedge \\
\quad (\forall\ \text{pd} : \text{PayDetails} \bullet \\
\qquad \text{req(pd)} \in \text{ether(w)} \Rightarrow \\
\qquad\quad \text{to(pd)} \in \text{purses(w)} \wedge \\
\qquad\quad \text{toSeqNo(pd)} < \text{nextSeqNo(purses(w)(to(pd))))} \wedge \\
\quad (\forall\ \text{pd} : \text{PayDetails} \bullet \\
\qquad \text{val(pd)} \in \text{ether(w)} \Rightarrow \\
\qquad\quad \text{to(pd)} \in \text{purses(w)} \wedge \text{ffrom(pd)} \in \text{purses(w)} \wedge \\
\qquad\quad \text{toSeqNo(pd)} < \text{nextSeqNo(purses(w)(to(pd)))} \wedge \\
\qquad\quad \text{fromSeqNo(pd)} < \text{nextSeqNo(purses(w)(ffrom(pd))))} \wedge
\end{array}
$$

Concrete Specification: World: invariant 2

$$
\begin{array}{l}
(\forall \ \mathrm{pd} : \mathrm{PayDetails} \bullet \\
\quad \mathrm{ack(pd)} \in \mathrm{ether(w)} \Rightarrow \\
\quad\quad \mathrm{to(pd)} \in \mathrm{purses(w)} \wedge \mathrm{ffrom(pd)} \in \mathrm{purses(w)} \wedge \\
\quad\quad \mathrm{ffrom(pd)} \in \mathrm{purses(w)} \wedge \\
\quad\quad \mathrm{toSeqNo(pd)} < \mathrm{nextSeqNo(purses(w)(to(pd)))} \wedge \\
\quad\quad \mathrm{fromSeqNo(pd)} < \mathrm{nextSeqNo(purses(w)(ffrom(pd)))} \wedge
\end{array}
$$

Concrete Specification: World: invariant 3

$$
\begin{array}{l}
(\forall \text{ pd} : \text{PayDetails} \bullet \\
\quad \text{pd} \in \text{fromLogs(w)} \Rightarrow \\
\qquad \text{req(pd)} \in \text{ether(w)} \wedge \\
\qquad \text{fromSeqNo(pd)} < \text{nextSeqNo(purses(w)(ffrom(pd)))} \wedge \\
\qquad \text{(status(purses(w)(ffrom(pd)))} \in \{\text{epr, epa}\} \Rightarrow \\
\qquad\quad \text{fromSeqNo(pd)} < \text{fromSeqNo(pdAuth(purses(w)(ffrom(pd))))))} \wedge \\
(\forall \text{ pd} : \text{PayDetails} \bullet \\
\quad \text{pd} \in \text{toLogs(w)} \Rightarrow \\
\qquad \text{req(pd)} \in \text{ether(w)} \wedge \\
\qquad \text{ack(pd)} \notin \text{ether(w)} \wedge \\
\qquad \text{(status(purses(w)(to(pd)))} \in \{\text{epv, eaTo}\} \Rightarrow \\
\qquad\quad \text{toSeqNo(pd)} < \text{toSeqNo(pdAuth(purses(w)(to(pd))))))} \wedge
\end{array}
$$

Concrete Specification: World: invariant 4

$$
\begin{array}{l}
(\forall\ \mathsf{n} : \mathsf{Name} \bullet \\
\quad \mathsf{n} \in \mathbf{dom}\ \mathsf{purses(w)} \wedge \mathsf{status(purses(w)(n))} = \mathsf{epa} \Rightarrow \\
\qquad \mathsf{req(pdAuth(purses(w)(n)))} \in \mathsf{ether(w))} \wedge \\
(\forall\ \mathsf{n} : \mathsf{Name} \bullet \\
\quad \mathsf{n} \in \mathbf{dom}\ \mathsf{purses(w)} \wedge \mathsf{status(purses(w)(n))} = \mathsf{epr} \Rightarrow \\
\qquad \mathsf{val(pdAuth(purses(w)(n)))} \notin \mathsf{ether(w)} \wedge \\
\qquad \mathsf{ack(pdAuth(purses(w)(n)))} \notin \mathsf{ether(w))} \wedge \\
(\forall\ \mathsf{n} : \mathsf{Name} \bullet \\
\quad \mathsf{n} \in \mathbf{dom}\ \mathsf{purses(w)} \wedge \mathsf{status(purses(w)(n))} = \mathsf{epv} \Rightarrow \\
\qquad \mathsf{req(pdAuth(purses(w)(n)))} \in \mathsf{ether(w)} \wedge \\
\qquad \mathsf{ack(pdAuth(purses(w)(n)))} \notin \mathsf{ether(w))} \wedge
\end{array}
$$

Concrete Specification: World: invariant 5

$$
\begin{aligned}
&(\forall\ \mathrm{pd} : \mathrm{PayDetails} \bullet \\
&\quad \mathrm{req(pd)} \in \mathrm{ether(w)} \wedge \mathrm{ack(pd)} \notin \mathrm{ether(w)} \Rightarrow \\
&\qquad (\mathrm{pd} \in \mathrm{toInEpv(w)} \vee \mathrm{pd} \in \mathrm{toLogs(w)})) \wedge \\
&(\forall\ \mathrm{pd} : \mathrm{PayDetails} \bullet \\
&\quad \mathrm{val(pd)} \in \mathrm{ether(w)} \wedge \mathrm{pd} \in \mathrm{toInEpv(w)} \Rightarrow \\
&\qquad \mathrm{pd} \in \mathrm{fromInEpa(w)} \vee \mathrm{pd} \in \mathrm{fromLogs(w)}) \wedge
\end{aligned}
$$

Concrete Specification: World: invariant 6

$$
\begin{aligned}
&(\forall\ \mathrm{pd} : \mathrm{PayDetails},\ \mathrm{n} : \mathrm{Name} \bullet \\
&\quad \mathrm{exceptionLogResult}(\mathrm{n}, \mathrm{pd}) \in \mathrm{ether}(\mathrm{w}) \Rightarrow \\
&\qquad \mathrm{n} \in \mathbf{dom}\ \mathrm{allLogs}(\mathrm{w}) \wedge \mathrm{pd} \in \mathrm{allLogs}(\mathrm{w})(\mathrm{n})) \wedge \\
&(\forall\ \mathrm{pds} : \mathrm{PayDetailsSet1},\ \mathrm{n} : \mathrm{Name} \bullet \\
&\quad \mathrm{exceptionLogClear}(\mathrm{n}, \mathrm{image}(\mathrm{pds})) \in \mathrm{ether}(\mathrm{w}) \Rightarrow \\
&\qquad \mathrm{n} \in \mathbf{dom}\ \mathrm{archive}(\mathrm{w}) \wedge \mathrm{pds} \subseteq \mathrm{archive}(\mathrm{w})(\mathrm{n})) \wedge \\
&(\forall\ \mathrm{m} : \mathrm{Message} \bullet \\
&\quad \mathrm{m} \in \mathrm{visible}(\mathrm{w}) \Rightarrow \mathrm{m} \in \mathrm{ether}(\mathrm{w}))
\end{aligned}
$$

14 conjuncts which must be proved as invariant for 11 operations!!

The Argument for Correctness

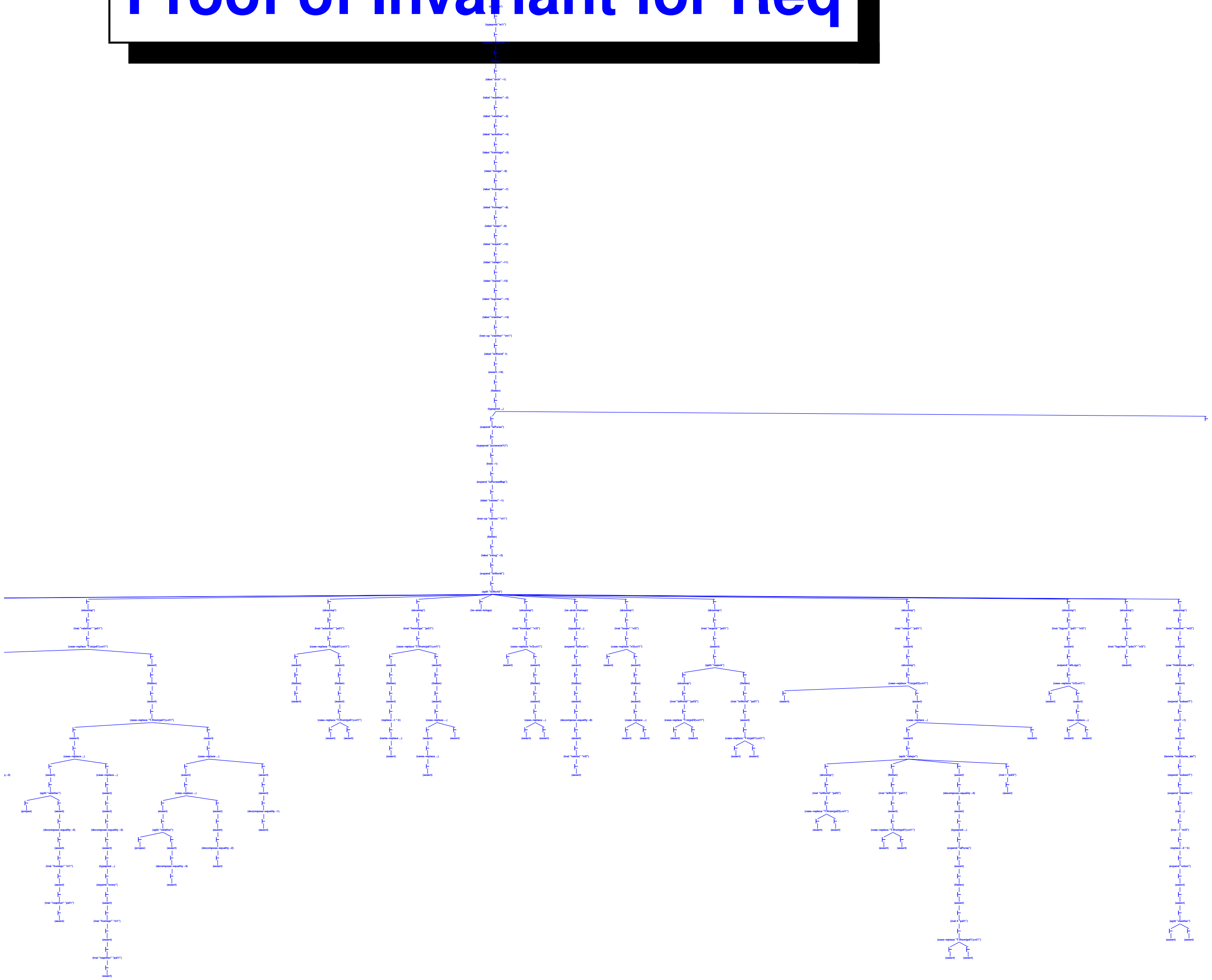
1. The abstract operations LeftTransfer, RightTransfer, Abort and No_op are *correct*; also sequence preserves correctness.
2. Each intermediate operation refines an abstract operation.
3. Each concrete operation refines the corresponding intermediate operation.

Easy!

Perhaps ...

- This is the 10th version of the specification, which is 2200 lines of RSL in 13 files.
- There are 366 proofs, perhaps half proved automatically.
- A typical invariant proof for the concrete specification is about 300 prover commands (recall there are 11 of these proofs).
- Other unpleasant proofs were that the concrete invariant implied the abstract one (150 prover commands), and that some sets defined by comprehension are finite.

Proof of Invariant for Req



Automation?

- The biggest problem is identifying the invariant.
  - Too strong: helps with proofs of refinement but can't be proved.
  - Too weak: easier to prove but refinement proofs fail.
- This problem has many large proofs with similar structure: tactics are worth developing.
  - The perfect tactic is very hard to write.
  - A tactic that does all the setting up of standard hypotheses, names them, and does the basic case analysis and tries to discharge the results can be very useful.
- One incautious `grind` generated (eventually) 1580 subgoals!

Did We Capture the Requirements Correctly?

- There may be many subtle points in 2200 lines of RSL!
- In the Z specification, for example, the description of a complete transfer is only informally stated, but seems to be unimplementable, because it requires you to know in advance a property of Abort that is underspecified (and perhaps nondeterministic): possible increase in nextSeqNo.
- Is there an "**axiom false**" somewhere?

Are our tools correct?

We rely on

- Translator from RSL to PVS
- PVS proof engine

Mondex in SAL

- Translated the concrete specification automatically from RSL to SAL.
- 8 versions to current one that runs with sal-smc (on a standard PC with 512MB of memory).
- Many changes to state structure, and reduced functionality.
- 3 versions used:
  - WORLD2 for correctness and liveness properties.
  - WORLD2INV for checking invariants.
  - CC version (special translation of WORLD2INV) for checking confidence conditions.

Why model check when you have a proof?

- Easier to do than proof.
- Could have found mistake in invariant in one version, and generally got more confidence that we were trying to prove things that are true.
- Can check confidence conditions, again before proof.
- Can show liveness properties, eg that a transfer is possible.

What was checked in SAL?

- Correctness:
  - All money is accounted.
  - The amount of money in circulation does not increase.
- World and purse invariants hold.
- Liveness in the sense that
  - An empty purse can become non-empty.
  - A non-empty purse can become empty.
  - Money can be lost.
- No confidence conditions violated.

but with only 2 purses and at most 3 transfers

Further work

- Drawing general conclusions for such systems:
  - modelling
  - checking
  - proving
- Can we improve the automation?
- Fix PVS 3.2 bug 894